Optimal Fiscal Policy With Heterogeneous Agents and Capital: Should We Increase or Decrease Public Debt and Capital Taxes?*

François Le Grand Xavier Ragot†

July 19, 2024

Abstract

We analyze optimal fiscal policy in a heterogeneous-agent model with capital accumulation and aggregate shocks, where the government uses public debt, a capital tax, and a progressive labor tax to finance public spending. We first study a tractable model and show that the steady-state optimal capital tax can be positive if credit constraints are occasionally binding. However, the existence of such an equilibrium depends on the shape of the utility function. We also characterize the optimal dynamic of public debt after a public spending shock. We confirm these findings by solving for optimal policy in a general heterogeneous-agent model.

Keywords: Heterogeneous agents, optimal fiscal policy, public debt

JEL codes: E21, H21, E44, D31.

*A very early version of this paper was circulated under the title "Optimal policy with heterogeneous agents and aggregate shocks." We thank Stefania Albanesi, Manuel Amador, Andy Atkeson, Adrien Auclert, Marco Bassetto, Anmol Bhandari, Javier Bianchi, Edouard Challe, Jonathan Heathcote, Greg Kaplan, Dirk Krueger, Albert Marcet, Juan Pablo Nicolini, and Gianluca Violante for comments and discussions. We thank Wouter den Haan, Christos Koulovatianos, Felix Kubler, Elisabeth Prohl, Morten Ravn, Vincent Sterk, Stephanie Schmitt-Grohe, Martin Uribe, Pierre-Olivier Weill, three anonymous referees, and participants in seminars at the European University Institute, Minneapolis Federal Reserve, National Bank of Belgium, New York University, University of Kyoto, University of Luxembourg, University of Zurich, as well as the EEA, NBER SI (MPF), SED, and T2M conferences. We also acknowledge financial support from ANR (ANR-20-CE26-0018 IRMAC).

†Le Grand: Rennes School of Business; francois.le-grand@rennes-sb.com. Ragot: SciencesPo-CNRS, OFCE, and CEPR; xavier.ragot@sciencespo.fr.

1 Introduction

What are the optimal levels of public debt and capital taxes? After a positive public spending shock, should the government temporarily increase capital taxes, or the progressivity of the tax system? These long-standing questions are likely to remain relevant in many countries in the coming years, as policymakers increasingly discuss additional public spending to combat climate change or for military purposes. Such questions require considering both the distortionary and redistributive effects of tax changes, while also taking into account general equilibrium effects. Heterogeneous-agent models in the tradition of the Bewley–Huggett–Imrohoroğlu–Aiyagari literature (Bewley, 1983; Imrohoroğlu, 1989; Huggett, 1993; Aiyagari, 1994; Krusell and Smith, 1998) are relevant tools for analyzing these questions, because they generate a realistic amount of heterogeneity along with general and dynamic equilibrium effects. However, after seminal papers investigating optimal fiscal policy in these environments (Aiyagari, 1995; Aiyagari and McGrattan, 1998), the literature has mainly moved towards a positive analysis. Little is known about the optimal levels and dynamics of public debt and capital taxes, because of both the theoretical and the computational difficulties of solving for optimal fiscal policy.

This paper analyzes optimal fiscal policy in heterogeneous-agent models, considering capital accumulation, progressive labor income taxation, capital taxation, public debt, and aggregate shocks. The only frictions considered are incomplete markets for idiosyncratic risk, occasionally binding credit constraints (which appear to be the key friction), and the given set of fiscal instruments. In particular, the planner in this model cannot use lump-sum taxes, which are known to potentially restore Ricardian equivalence in some settings (Bhandari et al., 2017). Ultimately, we find and characterize equilibria that feature optimal positive levels of both capital taxation and public debt. While our analysis admittedly abstracts from other frictions, such as nominal rigidities or frictional labor markets, we identify new mechanisms that will also be present in more general environments with these features.[1] Our paper makes three specific contributions, which we now discuss in more detail.

To understand the optimal steady-state levels of capital taxation and public debt, we first consider a simple heterogeneous-agent model and a utilitarian planner with

[1]Considering only price or wage stickiness would yield the same allocation as in our economy, since the only role of optimal monetary policy is price stability, given the set of fiscal instruments we consider (see LeGrand et al., 2022). Moreover, considering capital accumulation allows one to characterize the optimal dynamics of the capital tax and to discuss its relationship with the results of the vast Chamley–Judd literature on optimal capital taxation.

full commitment. In this simple model, two types of agents face deterministic income fluctuations between employment and unemployment, as in Woodford (1990). The only financial frictions are credit constraints, which can occasionally be binding, although there is no uninsurable idiosyncratic risk. Our first contribution is to prove that, contrary to the results of Judd (1985) and Chamley (1986), the steady-state optimal capital tax can be positive. However, it depends crucially on the shape of the utility function and on the intertemporal elasticity of substitution (hereafter IES). The intuition for this result is that savings induce a price externality, which the planner internalizes via a positive capital tax. The planner adjusts public debt so that the marginal return of capital equals the discount rate at the steady state. This is the well-known modified golden rule of Aiyagari (1995). With respect to savings, an extra unit saved increases the capital tax base and thus relaxes constraints on public finances in proportion to the capital tax. Absent any costs, the planner would like to set this marginal benefit to zero and maximize benefits – and hence to set a zero capital tax. Because the planner chooses savings that are consistent with the agents' Euler equations, the envelope theorem implies that there is no direct welfare effect for agents, and the costs on agents occur solely through a price externality of savings on interest rates and wages. When credit constraints are binding, in order to induce agents to save more, the planner would have to increase post-tax factor prices, and thus decrease the tax rate, what reduces the tax return. When setting aggregate savings, the planner trades-off the benefits in terms of larger fiscal base with the costs in terms of higher post-tax factor prices.

When the utility function is separable, the price externality is related to the elasticity of interest and wage rates to savings, which are pinned down by the Euler equation and the labor supply first-order condition. The savings externality is then found to be proportional to the gap between the inverse of the IES of employed and unemployed agents. This makes the shape of the utility function key to setting the optimal capital tax. In particular, the optimal capital tax is always zero in the standard case of a separable, Constant Relative Risk Aversion (CRRA) utility function, for which the gap between IESs is zero. This is consistent with the results of Chamley-Judd. In that case, the net effect of aggregate savings on post-tax prices is null and there is no savings externality. The planner can simply choose savings that maximize its resources, and hence maintain a capital tax of zero. This result is also consistent with the claim of Chien and Wen (2023) and the numerical investigation of Auclert et al. (2022).

However, the steady-state capital tax can be positive if the utility function deviates from a constant IES. This is the case, for example, with separable DRRA (e.g., Stone-Geary or

Fishburn) utility functions. For non-separable utility functions, the elasticity of the factor prices with respect to savings includes an additional term related to the cross-derivative of the utility function with respect to consumption and labor supply. This new term explains why the optimal capital tax is positive for the Greenwood-Hurcowitz-Huffman (GHH) or the King-Plosser-Rebelo (KPR) utility functions.

Our second contribution is to fully characterize the conditions for the existence of an equilibrium in this environment with both positive optimal capital taxation and positive optimal public debt. To study existence, we consider a GHH-type utility function, such as in Aiyagari (1995), Diamond (1998) or Açikgöz et al. (2022). The existence of the steady-state equilibrium relies on three independent conditions: a non-first-best condition, a so-called Straub–Werning condition, and a standard Laffer condition. The Straub–Werning condition is based on Straub and Werning (2020) and states that public spending must be low enough to ensure a stationary steady state, and to avoid a situation in which the planner chooses to continuously reduce the capital stock (despite being able to raise enough resources in the steady state). Finally, a fourth, Blanchard–Kahn condition, ensures the stability of the equilibrium. Alongside a positive capital tax, the optimal fiscal system can be characterized (for some parameter combinations) as featuring a positive public debt, when saving is higher than the optimal capital stock.We extend this simple setting to an economy with ex-ante heterogeneity and show that the optimal capital tax depends on the social weights of the Ramsey planner. This result again differs from that of Judd (1985).

Equipped with these results, we analyze the optimal dynamics of fiscal policy after a one-time positive shock to public spending (a so-called MIT shock). Our third contribution is to show that, for a given net present value (NPV) of public spending, public debt increases (resp. decreases) when the persistence of the shock is low (resp. high). Consequently, the persistence of the shock is a key driver of the optimal dynamics of public debt. The intuition for these results is the following. In contrast to the complete-market case where agents initially hold some capital, in the incomplete-market model, the capital tax is not used to fully front-load the adjustment because taxing capital reduces the ability of agents to self-insure when markets are incomplete. In addition, in this type of model, public debt converges to its optimal steady-state value for any transitory shock to public spending. Consequently, if the shock's persistence is high, a transitory increase in public debt would require a welfare-reducing, highly persistent increase in taxes to finance public spending and reduce public debt. Therefore, the optimal policy is to front-load the adjustment and to temporarily reduce the public debt. When persistence is low, in contrast, the increase in public debt improves consumption smoothing, and a small increase in taxes is sufficient to ensure that public debt converges. The claim that optimal public debt can fall after a persistent public spending shock is already made in Feldstein (1985), who introduced a quadratic tax adjustment cost. Compared to this seminal literature, the current paper provides a micro-foundation for the cost of tax changes, based on distributional considerations, and generates an optimal long-run level of public debt.

Finally, we verify that the previous results, obtained in a stylized model, still hold in a realistic quantitative model. In this model, ex-ante different types of agents, all endowed with a GHH utility function, face heterogeneous uninsurable income risk. The planner aims to finance public spending through a capital tax, a nonlinear labor tax à la Heathcote et al. (2017), and public debt. The planner's Social Welfare Function (SWF) assigns weights to agents that depend on their ex-ante type. Our quantitative strategy is, first, to use an inverse-optimal approach to identify the SWF weights from the observed fiscal system, which is assumed to be an optimal steady state (as in Heathcote and Tsujiyama, 2021 among others). Second, we verify that the existence conditions identified in the analytical model also hold in the general model. Third, using the identified SWF, we compute the optimal dynamics of the capital tax, the labor tax, progressivity, and public debt after an (MIT) public spending shock. This strategy allows us to simulate the dynamics around a quantitatively relevant steady state. The results of the quantitative model are consistent with those of the theoretical model. Public debt increases when the persistence of the public spending shock is low, and decreases otherwise. The quantitative model also provides additional results. The optimal progressivity of the labor and the capital tax both increase after a positive public spending shock, but the increase is smaller when the persistence of the shock is higher. Optimal public debt also exhibits persistent deviations that are quantitatively much larger than that of other variables.

This paper is related to the literature on optimal fiscal policy in heterogeneous-agent models.[2] As mentioned above, the existence of well-defined steady-state Ramsey equilibria is still an open question. Conesa et al. (2009) considered transitions with constant instruments. Chien and Wen (2023) and Auclert et al. (2022) find that the Ramsey steady-state equilibrium does not exist for separable CRRA utility functions. Dyrda and Pedroni (2022) quantitatively solved for optimal policy by considering the full path of the policy instruments and using a KPR utility function. Aiyagari (1995) and Açikgöz et al. (2022) analyze optimal public debt when there is no wealth effect on labor supply. Bassetto and Cui (2021) study an environment where public debt can relax the producer's

[2] A large literature provides a positive analysis of fiscal policy in heterogeneous-agent models (e.g., Floden, 2001; Heathcote, 2005; Rohrs and Winter, 2017; Ferriere and Navarro, 2023, among many others).

credit constraint. They find that optimal steady-state capital taxes are positive, when public debt is constrained to be at the top of the Laffer curve. Our results show that these apparently contradictory results are consistent, as they are based different utility functions.[3]

Analyzing optimal fiscal policy in such an environment obviously relies heavily on results about idiosyncratic risk in complete-market economies.[4] As mentioned above, incomplete-market models allow for consideration of optimal positive steady-state capital taxation and redistribution. A recent literature reports the development of tools for solving for optimal policies with heterogeneous agents involving mostly monetary policy, for which the steady-state allocation is simpler to characterize, as optimal inflation is null (e.g., Bhandari et al., 2021; Acharya et al., 2023; LeGrand et al., 2022; Nuño and Thomas, 2022, among others). We use the truncation approach of LeGrand and Ragot (2022a), using the refinement of LeGrand and Ragot (2022b) to solve the curse of dimensionality. This method builds on the factorization method introduced by Marcet and Marimon (2019) and allows one to easily simulate models with many instruments and aggregate shocks. Because it is relatively new, we summarize the method in Section 4 below.

The rest of this paper is organized as follows. In Section 2, we present the general environment. In Section 3, we present simplifying assumptions and solve the tractable model. We present the general model and derive optimality conditions in Section 4. In Section 5 we calibrate and simulate the general model. We conclude in Section 6.

2 The Environment

Time is discrete and indexed by $t = 0, 1, \ldots$, and the economy is populated by a continuum of F heterogeneous types of agents. The type of an agent determines the dynamics of the productivity risk it faces. Each type f is distributed along a set I^f with measure ℓ^f. We follow Green (1994) and assume that the law of large numbers holds. There is a share m^f of type f, where $\sum_{f=1}^{F} m^f = 1$ and the population of each type is one, such that $\sum_{f=1}^{F} m^f \int_i \ell^f(di) = 1$.

Furthermore, the economy features production and a benevolent government that raises

[3] Albanesi and Armenter (2012) provide general sufficient conditions for the optimal steady-state capital tax to be zero in many environments. These conditions are not fulfilled in our setup for relevant cases, because the planner would need to use a distorting labor tax to finance public spending when the capital tax is zero, preventing the economy from converging to the first-best allocation.

[4] For relevant contributions, see Barro (1979); Chari et al. (1994); Farhi (2010); Bassetto (2014); Chari et al. (2020); Straub and Werning (2020); Collard et al. (2023) among others.

distorting taxes and public debt to finance an exogenous stream of public spending.

Risks. The aggregate shock solely affects public spending, denoted by $(G_t)_{t\geq 0}$, and is therefore assimilated to a public spending shock. We discuss below, in Section 5.4, the outcome of the model for other shocks. Furthermore, we assume that the whole path of public spending $(G_t)_{t\geq 0}$ becomes known to all agents in period 0. We will solve for the optimal adjustment of the economy after such a shock (often called an MIT shock), assuming that the planner cannot renege on their past commitments (See Section 4.4 below for further discussion).

Each agent's type f differs according to its productivity process. Each productivity process is a first-order Markov chain characterized by a finite set of productivity levels $\mathcal{Y}^f$ and a transition matrix Π^f. For the sake of simplicity, we assume that the number of possible productivity levels is the same for all types, and denoted by J – such that all transition matrices have the same dimension $J \times J$. We assume that each productivity process admits a unique stationary distribution that is denoted by the vector S_y^f, verifying $S_y^f = (S_y^f)^\top \Pi^f$.[5] In period t, the productivity of agent i of type f is $y_{i,t}^f$ and they will earn a before-tax labor wage $\tilde{w}_t y_{i,t}^f l_{i,t}^f$, where $l_{i,t}^f$ denotes their labor supply and $\tilde{w}_t$ the before-tax hourly wage. Their whole history of shocks up to t is denoted by $y_i^{f,t} := \{y_{i,0}^f, ..., y_{i,t}^f\}$.

Production. The production sector is standard. The consumption-investment goods of the economy are produced by a profit-maximizing representative firm. At any date t, the firm's production function combines labor L_t and capital K_{t-1}—which must be installed one period in advance—to produce Y_t units of the consumption goods. The production function is assumed to be of the Cobb-Douglas type, featuring constant returns to scale and capital depreciation. The total factor productivity is normalized to one. Formally, net-of-depreciation production is defined as

$$Y_t = F(K_{t-1}, L_t) = K_{t-1}^\alpha L_t^{1-\alpha} - \delta K_{t-1},$$

where $\alpha \in (0, 1)$ is the capital share and $\delta \in (0, 1)$ is the capital depreciation rate.

The firm rents labor and capital at respective factor prices $\tilde{w}_t$ and $\tilde{r}_t$. The profit maximization conditions of the firm imply the following expressions for factor prices:

$$\tilde{w}_t = F_{L,t} \text{ and } \tilde{r}_t = F_{K,t}, \tag{1}$$

[5] In the quantitative analysis of Section 4, the Markov chain can be shown to be irreducible and aperiodic; hence S_y^f is known to exist and to be unique.

where we use $F_{L,t} := F_L(K_{t-1}, L_t)$ and $F_{K,t} := F_K(K_{t-1}, L_t)$ to lighten notations.

Assets. In addition to capital, the economy also features public debt, whose size is denoted by B_t in period t. Public debt consists of one-period bonds issued by a benevolent government, which are assumed to be default-free. Because of our assumption of MIT shocks, there is no aggregate risk in this economy. Both capital and public debt are thus perfect substitutes, and no-arbitrage implies that they must pay the same after-tax return. Agents' savings, denoted by $(a^f_{i,t})_{i,f}$ at date t, are restricted to remain greater than an exogenous limit $-\underline{a} \leq 0$.

Period 0. We assume that the economy starts in period -1 with an endowment of wealth and productivity $(a^f_{i,-1}, y^f_{i,0})_i$ drawn from a distribution Λ_0, a given amount of public debt B_{-1}, and a given amount of capital K_{-1}, which together satisfy $K_{-1} + B_{-1} = \sum_{f=1}^{F} m^f \int_i a^f_{i,-1} \ell(di)$. The MIT shock is the public spending path $(G_t)_{t\geq 0}$, which is revealed at period -1 before households actually perform their portfolio choice. As a consequence, and as there is no aggregate risk, no arbitrage implies that the two assets must have the same after-tax return in all periods, including period 0. The before-tax real interest rate between period -1 and period 0 is denoted $\tilde{r}_0$, and the MIT shock affects the allocation from period 0 onward.

Government. A benevolent government has to finance the exogenous stream of public spending $(G_t)_{t\geq 0}$ by levying distorting taxes on capital and labor, and by issuing public debt. The tax on capital is linear with a rate $(\tau^K_t)_{t\geq 0}$, and is actually levied on all interest bearing assets (capital and public debt). The tax on labor income is assumed to be nonlinear and possibly time-varying. We denote by $T_t(\tilde{w}yl)$ the amount of labor tax paid at date t by an agent earning the pre-tax labor income $\tilde{w}yl$. We follow Heathcote et al. (2017) (hereinafter HSV) and consider the following functional form:

$$T_t(\tilde{w}yl) := \tilde{w}yl - \kappa_t(\tilde{w}yl)^{1-\tau_t}, \tag{2}$$

where κ_t captures the level of labor taxation and τ_t the progressivity. Both parameters are assumed to be time-varying and will be the planner's instruments in the general model. When $\tau_t = 0$, labor tax is linear with rate $1 - \kappa_t$; oppositely, $\tau_t = 1$ corresponds to full income redistribution, where all agents earn the same post-tax income κ_t. Functional form (2) combined with the linear capital tax allows one to realistically reproduce the actual US system and its progressivity (see Heathcote et al., 2017 or Ferriere and Navarro, 2023).[6]

Combining the above elements, the government budget constraint can be written as:

$$G_t + (1+\tilde{r}_t)B_{t-1} \leq \sum_{f=1}^{F} m^f \int T_t(\tilde{w}_t y^{i,f} l_{i,t}^f)\ell^f(di) + \tau_t^K \tilde{r}_t(B_{t-1} + K_{t-1}) + B_t. \qquad (3)$$

This states that public spending and past public debt repayment can be financed out of the proceedings of labor and capital taxation, as well as by the issuance of new public debt. To simplify the government budget constraint, in the spirit of Chamley (1986) we introduce generalized post-tax factor prices, which are denoted without a tilde. We define the gross and net interest rates r_t and R_t, respectively, and the wage rate w_t as:

$$w_t := \kappa_t(\tilde{w}_t)^{1-\tau_t}, \qquad (4)$$

$$R_t := 1 + r_t = 1 + (1-\tau_t^K)\tilde{r}_t. \qquad (5)$$

The model can be expressed analytically using the pair of post-tax rates (R_t, w_t) rather than pre-tax ones $(\tilde{r}_t, \tilde{w}_t)$, which simplifies the algebra. The values of the fiscal instruments τ_t^K, κ_t, and τ_t can then be recovered from the allocation. Using the constant return-to-scale property of the production function, the governmental budget constraint (3) becomes:

$$G_t + R_t B_{t-1} + w_t \sum_{f=1}^{F} m^f \int_i (y_{i,t}^f l_{i,t}^f)^{1-\tau_t}\ell^f(di) \leq F(K_{t-1}, L_t) - (R_t - 1)K_{t-1} + B_t, \qquad (6)$$

which can be interpreted by observing that total output and new public debt are used to finance public spending, past public debt repayment, post-tax capital rents, and post-tax wages. In the constraint (6), the effect of factor supplies on aggregate output is fully internalized by the government. Equation (6) can indeed be interpreted by viewing the planner as having the economy's output and newly issued public debt as revenue, and paying back old debt, public spending, and factor supplies with post-tax rates. This has two implications, which matter for the discussion of the planner's choices in Section 3.1. First, it explains why zero capital tax, which maximizes output, also relaxes the budget constraint the most. Indeed, if everything else were constant, including R_t, the level of the capital stock that maximizes the right hand side of (6) would satisfy $1 + F_{K,t} = R_t$, which corresponds to zero capital tax – as can be seen from equations (1) and (5). Second,

[6] The literature uses either the combination of a linear tax and a lump-sum transfer (e.g., Açikgöz et al., 2022; Dyrda and Pedroni, 2022) or the HSV structure. Heathcote and Tsujiyama (2021) showed that the HSV structure is quantitatively more relevant. However, we show in Appendix I that our results still hold in the presence of an affine tax system.

equation (6) also shows that, if everything else were constant, higher post-tax interest and wage rates have a negative impact on the governmental budget constraint. The planner would indeed have to give more resources to agents, and have less resources to finance public spending.

Agents' program and resource constraints. At each date t, agents consume goods in quantity c_t, supply labor in quantity l_t, and save an amount a_t. They derive an instantaneous utility from consumption and labor supply denoted by $U(c_t, l_t)$; the utility function will be specified later. The discount factor is constant and denoted by $\beta \in (0, 1)$.

Using the post-tax rate definition (4), the post-tax labor income of an agent i of type f amounts to $\tilde{w}_t y^f_{i,t} l^f_{i,t} - T_t(\tilde{w}_t y^f_{i,t} l^f_{i,t}) = w_t (y^f_{i,t} l^f_{i,t})^{1-\tau_t}$, while post-tax capital income is equal to $R_t a_{i,t-1}$. Formally, the program of agent i of type f endowed with the given initial wealth $a^f_{i,-1}$ can be expressed as:

$$\max_{\{c^f_{i,t}, l^f_{i,t}, a^f_{i,t}\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c^f_{i,t}, l^f_{i,t}), \tag{7}$$

$$c^f_{i,t} + a^f_{i,t} = R_t a^f_{i,t-1} + w_t (y^f_{i,t} l^f_{i,t})^{1-\tau_t}, \tag{8}$$

$$a^f_{i,t} \geq -\underline{a},\ c^f_{i,t} \geq 0,\ l^f_{i,t} \geq 0. \tag{9}$$

Note that because of our assumption of MIT shocks, the expectation operator in (7)—as well as in the rest—solely concerns idiosyncratic shocks. (8) is the budget constraint, and inequalities (9) are the credit constraint and the non-negativity constraints.

The solution of the previous program is a set of policy rules defined over the product space of productivity histories and initial asset holdings: $c^f_t : (\mathcal{Y}^f)^t \times [-\bar{a}; +\infty) \to \mathbb{R}^+$, $a^f_t : (\mathcal{Y}^f)^t \times [-\bar{a}; +\infty) \to [-\bar{a}; +\infty)$, and $l^f_t : (\mathcal{Y}^f)^t \times [-\bar{a}; +\infty) \to \mathbb{R}^+$. To lighten the notation, we will simply write $c^f_{i,t}$, $a^f_{i,t}$, and $l^f_{i,t}$ (instead of $c^f_t(y^{f,t}_i, a^f_{i,-1})$, $a^f_t(y^{f,t}_i, a^f_{i,-1})$, and $l^f_t(y^{f,t}_i, a^f_{i,-1})$) and use the same notation for all variables.[7]

Denoting by $\beta^t \nu^f_{i,t} \geq 0$ the Lagrange multiplier on the agent's credit constraint, the consumption Euler equation can be written as

$$U_c(c^f_{i,t}, l^f_{i,t}) = \beta \mathbb{E}_t \left[R_t U_c(c^f_{i,t+1}, l^f_{i,t+1}) \right] + \nu^f_{i,t}, \tag{10}$$

where we denote by U_c and U_l the first-order derivatives with respect to c and l, and by

[7] Hence, the aggregation of the variable X_t in period t will be written as $\int_i X^f_{i,t} \ell^f(di)$ instead of the more involved explicit notation $\int_{a^f_{-1}} \sum_{y^{f,t} \in \mathcal{Y}^t} \theta^f_t(y^{f,t}) X(y^{f,t}, a^f_{-1}) d\Lambda_0(a^f_{-1}, y^f_0)$, where $\theta^f_t(y^{f,t})$ is the probability of the occurrence of history $y^{f,t}$ in period t for an agent of type f.

U_{xy} with $(x, y = c, l)$ the second-order derivatives.

The first-order condition (FOC) on labor is:

$$-U_l(c_{i,t}^f, l_{i,t}^f) = (1 - \tau_t) w_t y_{i,t}^f (y_{i,t}^f l_{i,t}^f)^{-\tau_t} U_c(c_{i,t}^f, l_{i,t}^f), \tag{11}$$

and the clearing of financial and labor markets implies the following equalities:

$$A_t = K_t + B_t \text{ and } \sum_{f=1}^{F} m^f \int y_{i,t}^f l_{i,t}^f \ell^f(di) = L_t. \tag{12}$$

The clearing of the goods market reflects the fact that the sum of aggregate consumption, public spending, and the new capital stock balances the production output and past capital:

$$\sum_{f=1}^{F} m^f \int_i c_{i,t}^f \ell^f(di) + G_t + K_t = K_{t-1} + F(K_{t-1}, L_t). \tag{13}$$

The Social Welfare Function. The planner considers a weighted sum of agents' intertemporal utilities, where the social weight of each agent (sometimes referred to as Negishi or Pareto weights) is denoted by ω^f and depends solely on their time-invariant type. The utilitarian case corresponds to $\omega^f = 1$ for all f. The aggregate social welfare W_0 can thus be written as:

$$W_0 = \sum_{f=1}^{F} m^f \omega^f \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_{i \in I^f} U(c_{i,t}^f, l_{i,t}^f) \ell^f(di) \right), \tag{14}$$

where the term between brackets is the sum, over the whole population I^f, of the ex-ante intertemporal utilities of agent i of type f. The planner attributes a social weight ω^f to each agent, corresponding to the agent's type. In addition, the sum of the weighted utilities of type f agents is itself weighted by the share m^f of type f in the total population. We normalize the social weights to sum to 1: $\sum_{f=1}^{F} m^f \omega^f = 1$.

Equilibrium definitions The Ramsey problem with full commitment consists in finding the fiscal policy that delivers the competitive equilibrium with the highest aggregate social welfare. We start with the formal definition of a competitive equilibrium.

Definition 1 (Competitive equilibrium (CE))**.** *A competitive equilibrium is a collection of individual variables* $(c_{i,t}^f, l_{i,t}^f, a_{i,t}^f)_{t\geq 0}^{i\in\mathcal{I}^f, f=1,\ldots,F}$*, aggregate quantities* $(K_t, L_t, Y_t)_{t\geq 0}$*, prices* $(\tilde{w}_t, \tilde{r}_t)_{t\geq 0}$*, fiscal policy* $(\tau_t^K, \kappa_t, \tau_t, B_t)_{t\geq 0}$*, and public spending* $(G_t)_{t\geq 0}$ *such that for an initial distribution of wealth and productivity* $(a_{i,-1}^f, y_{i,0}^f)_{i\in\mathcal{I}^f, f=1\ldots F}$ *and for initial values of* *capital stock and public debt verifying* $K_{-1} + B_{-1} = \sum_{f=1}^{F} m^f \int_i a_{i,-1}^f \ell^f(di)$*, the following holds. i) Given prices, individual strategies* $(c_{i,t}^f, l_{i,t}^f, a_{i,t}^f)_{i,t\geq 0}^f$ *solve the agent's optimization program in equations (7)–(9). ii) Financial, labor, and goods markets clear: for any* $t \geq 0$*, equations (12) and (13) hold. iii) The government budget is balanced: equation (3) holds for all* $t \geq 0$*. iv) The pre-tax factor prices* $(\tilde{w}_t, \tilde{r}_t)_{t\geq 0}$ *are consistent with the firm's program (1).*

Using the previous definition, we now state the formal definition of the Ramsey equilibrium, and the stationary Ramsey equilibrium.

Definition 2 (Ramsey Equilibrium (RE))**.** *A Ramsey Equilibrium is a competitive equilibrium, which generates the highest welfare, measured by* W_0*, over the set of fiscal policies* $(\tau_t^K, \kappa_t, \tau_t, B_t)_{t\geq 0}$ *satisfying the governmental budget constraint.*

Definition 3 (Stationary Ramsey Equilibrium (SRE))**.** *A stationary Ramsey equilibrium is a Ramsey equilibrium for which aggregate quantities* $(K_t, L_t, Y_t)_{t\geq 0}$*, prices* $(\tilde{w}_t, \tilde{r}_t)_{t\geq 0}$*, fiscal policy* $(\tau_t^K, \kappa_t, \tau_t, B_t)_{t\geq 0}$*, and public spending* $(G_t)_{t\geq 0}$ *are constant.*

First-best allocation. A natural candidate against which to compare the outcome of the Ramsey equilibrium is the first-best allocation. The latter is the solution of the program maximizing aggregate social welfare W_0, subject only to the resource condition. Formally, it solves the following program:

$$\max_{((c_{i,t}, l_{i,t})_{i\in I}, L_t, K_t)_{t\geq 0}} W_0, \tag{15}$$

$$\text{s.t. } \sum_{f=1}^{F} m^f \int_i c_{i,t}^f \ell^f(di) + G_t + K_t = K_{t-1} + F(K_{t-1}, L_t), \tag{16}$$

$$\text{and } \sum_{f=1}^{F} m^f \int y_{i,t}^f l_{i,t}^f \ell^f(di) = L_t, \ K_{-1} \text{ given.}$$

The solution of this program provides the Pareto frontier of this economy by varying the social weights ω^f.

3 Analyzing Existence and Dynamics in a Simple Model

We first study optimal fiscal policy in a simple model, in which we can derive analytical results. The main simplifying assumption is to consider deterministic productivity fluctuations, as introduced by Woodford (1990). The gain of this approach is that it yields analytical solutions—including a characterization of the Ramsey allocation—but also that it provides the proof that positive optimal capital taxation and public debt are the results of credit constraints, and not of incomplete insurance markets.

The simplifying assumptions introduced in the environment of Section 2 are as follows.

Assumption A.

1. *The labor tax is linear: in (2) we set* $\tau_t = 0$ *and denote* $\tau_t^L := 1 - \kappa_t$ *such that* $T_t(\tilde{w}yl) := \tau_t^L \tilde{w}yl$.
2. *The credit constraint is set to zero:* $\underline{a} = 0$.
3. *There is only one productivity process (*$F = 1$ *and* $m_1 = 1$*), which can take only two values:* 0 *and* 1*. The transition matrix is anti-diagonal:* $\Pi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ *and there is initially a unit mass of agents in each productivity level.*

In this setup, the planner is endowed with three instruments: a linear capital tax, a linear labor tax, and public debt. Agents face deterministic productivity variations, which they can smooth using their savings, subject to borrowing limits. Due to the assumption about the initial distribution and the anti-diagonal transition matrix, the total population has been renormalized to 2 and in every period there is a population of mass one that is employed (called "employed", with subscript e) and another that is unemployed (called "unemployed", with subscript u).

The remainder of this section is organized as follows. In Section 3.1, we characterize the planner's first-order conditions for general utility functions, and then discuss different standard cases (CRRA, DRRA, CARA, GHH, KPR, among others). In Section 3.2, we focus on the GHH utility function and derive formal existence conditions. In Section 3.3, we provide our main results about the dynamics of public debt in the simple model. Finally, in Section 3.4, we relax the assumption of a homogeneous population and study the effects of the SWF weights on the equilibrium capital tax.

3.1 Characterizing the Planner's FOCs in an SRE

As a preliminary remark, observe that in any non-trivial equilibrium, employed agents cannot be credit-constrained at any date: otherwise unemployed agents would consume zero, as they do not earn any labor income. Thus, there are only two possible steady-state equilibria: one in which unemployed agents are not credit-constrained, and one in which they are. Thus, the Ramsey program can be written as follows:

$$\max_{(c_{e,t},c_{u,t},a_{e,t},a_{u,t},l_{e,t}B_t,A_t,R_t,w_t)} \sum_{t=0}^{\infty} \beta^t \Big(U(c_{e,t}, l_{e,t}) + U(c_{u,t}, 0) \Big) \tag{17}$$

$$\text{s.t. } c_{e,t} + a_{e,t} = R_t a_{u,t-1} + w_t l_{e,t}, \tag{18}$$

$$c_{u,t} + a_{u,t} = R_t a_{e,t-1}, \tag{19}$$

$$U_c(c_{e,t}, l_{e,t}) = \beta R_{t+1} U_c(c_{u,t+1}, 0), \tag{20}$$

$$U_c(c_{u,t}, 0) \geq \beta R_{t+1} U_c(c_{e,t+1}, l_{e,t+1}), \text{ with equality if } a_{u,t} > 0, \tag{21}$$

$$-U_l(c_{e,t}, l_{e,t}) = w_t U_c(c_{e,t}, l_{e,t}), \tag{22}$$

$$F(A_{t-1} - B_{t-1}, l_{e,t}) + B_t \geq G_t + B_{t-1} + (R_t - 1)A_{t-1} + w_t l_{e,t}, \tag{23}$$

$$A_t = a_{e,t} + a_{u,t}, \tag{24}$$

$$a_{e,t}, a_{u,t} \geq 0, \tag{25}$$

$$c_{e,t}, c_{u,t} > 0 \text{ and } l_{e,t}, l_{u,t} \geq 0. \tag{26}$$

The planner maximizes the aggregate welfare criterion (17) subject to the following: the constraints (18)–(22), which guarantee the optimality of individual choices (budget constraints, Euler equations, and labor FOC, respectively); the governmental budget constraint (23); the financial market clearing condition (24); the credit constraints (25); and the positivity constraints (26).

SRE with non-binding credit constraints. When credit constraints do not bind, we recover the seminal result of Chamley (1986) and Judd (1985), that the optimal capital tax is zero in an SRE. Indeed, on the one hand, using the two Euler equations (20) and (21), we find $\beta R = 1$. On the other hand, the planner's FOC for public debt implies the modified golden rule at the steady-state: $\beta(1 + \tilde{r}) = 1$. We deduce that post- and pre-tax rates must coincide: $R = 1 + \tilde{r}$ and the capital tax is null: $\tau^K = 0$.[8] A positive capital tax at the steady state can only be optimal in an equilibrium with binding credit constraints.

SRE with binding credit constraints. We characterize the equilibrium where the credit constraints bind for unemployed agents ($a_{u,t} = 0$ for all t). Since unemployed agents are credit constrained, the employed and unemployed budget constraints (18) and (19) simplify into $c_{e,t} = w_t l_{e,t} - a_{e,t}$ and $c_{u,t} = R_t a_{e,t-1}$.

[8] We here characterize a SRE. Its existence is however not ensured in all cases, as shown by Straub and Werning (2020), and as we discuss below.

To summarize the effects at stake when the planner optimally sets the capital and labor taxes, we first define the following quantities:

$$\sigma_e := -c_e \frac{U_{cc}(c_e, l_e)}{U_c(c_e, l_e)},\ \sigma_u := -c_u \frac{U_{cc}(c_u, 0)}{U_c(c_u, 0)}, \tag{27}$$

$$\varphi_e := \left(l_e \frac{U_{ll}(c_e, l_e)}{U_l(c_e, l_e)} \right)^{-1},\ \varsigma_{c,e}^{l} := l_e \frac{U_{cl}(c_e, l_e)}{U_c(c_e, l_e)},\ \varsigma_{l,e}^{c} := c_e \frac{U_{cl}(c_e, l_e)}{U_l(c_e, l_e)}. \tag{28}$$

The quantities $\sigma_e, \sigma_u \geq 0$ in equation (27) are the inverse of the intertemporal elasticity of substitution (IES) for employed and unemployed agents. The inverse of the IES is also, by analogy to the static case, referred to as the relative risk aversion (RRA), even though in our case there is no risk and the correct interpretation is in terms of an elasticity. The IES is constant for CRRA utility functions, and hence identical for both agents independently of consumption and labor choices. The quantity $\varphi_e \geq 0$ is the Frisch elasticity of the labor supply for employed agents. Finally, $\varsigma_{c,e}^{l}$ is the elasticity of the marginal utility of consumption of employed agents, $U_c(c_e, l_e)$, to labor supply, while $\varsigma_{l,e}^{c}$ is the elasticity of the marginal utility of labor supply of employed agents, $U_l(c_e, l_e)$, to consumption. These two last terms are null when the utility function U is separable in consumption and labor. The next Proposition first presents a characterization of an SRE.

Proposition 1. *In any standard SRE with a binding credit constraint for unemployed households, we have:*

1. $1 + F_K = \frac{1}{\beta}$;
2. *The post-tax interest and wage rates satisfy:*

$$\underbrace{1 - \beta R}_{\text{Smoothing wedge}} = \underbrace{\frac{F_L - w}{w}}_{\text{Labor wedge}} \underbrace{\frac{\sigma_u - \sigma_e + \varsigma_{c,e}^{l}}{\sigma_e + \frac{1}{\psi_e} - \varsigma_{c,e}^{l} + \varsigma_{l,e}^{c}}}_{\text{Net Distributional Gain}}. \tag{29}$$

The proof can be found in Appendix F.1.[9] Before commenting on the equality (29), two remarks are in order. The first part of the proposition has been well known since Aiyagari (1995), and is called the modified golden rule. Since the government faces no credit constraint, it is optimal to set the public debt so that the marginal productivity

[9]We refer to a standard SRE as an equilibrium where Lagrange multipliers are finite. We characterize the existence of these equilibria in Proposition 3.2.1 below. Lansing (1999) has shown that in the special case of an IES of 1, a SRE could exist in a representative agent model, with diverging Lagrange multipliers. This is not the case in our model as shown in Appendix C.8.

of capital equals the discount rate. As noted above, this depends solely on the planner's ability to adjust public debt freely, and is independent of whether or not credit constraints are binding for private agents. A second remark is that condition (29) is a necessary condition of the SRE when credit constraints are binding for unemployed agents. This is a first step towards characterizing the existence of such equilibria, which is not ensured at this stage.

To better understand the naming of terms and the intuitions in equation (29), it is worth considering the planner's optimality conditions. Different perspectives on the planner's optimization program are possible; however, the simplest way to understand the relationship (29) between post-tax interest and wage rates is to think of the planner as jointly setting savings and the labor supply, while internalizing the general equilibrium effects (through prices) of its choices. First, the FOC associated with the savings choice can be written as:

$$1 - \beta R = \Xi \times \left(\sigma_u - \sigma_e + \varsigma^l_{c,e}\right), \tag{30}$$

where $\Xi \geq 0$ is a product of various Lagrange multipliers summarizing equilibrium distortions. When the planner sets the level of savings, equation (30) shows that the planner trades off the marginal benefit of higher savings, implying a larger tax base (on the left hand side), with the related marginal cost that channels through higher post-tax interest and wage rates (on the right hand side). The marginal benefit is proportional to the capital tax, since an extra unit of saving increases the capital tax base. Indeed, because of the modified golden rule, we have: $1-\beta R = \beta(1+F_K-R) = \beta(\tilde{r}-r) = (1-\beta)\tau^K$. When the capital tax is positive, $\beta R < 1$ and the Euler equation implies imperfect consumption smoothing. Thus, the term $1-\beta R$ is called a "smoothing wedge." Absent any costs, the planner would set savings so as to maximize the benefit for its resources – and hence set the marginal benefit to zero. This would imply a zero capital tax and $1 + F_K = R$, as discussed after equation (6).

The planner does not maximize the benefit of savings for its resources because of the externality of savings that tends to increase post-tax interest and wage rates, which is detrimental for the planner's resources as explained after equation (6). Note that there is no direct welfare effect of this extra saving, due to the envelope theorem. By the construction of the Ramsey program, the planner chooses savings that are optimal for households and thus consistent with the Euler equation. The externality of savings on prices channels through the agents' consumption. Extra savings raise the consumption of the unemployed and reduce the consumption of employed. Both effects contribute to raise the post-tax interest rate via the Euler equation (20). However, the lower consumption of the employed in general decreases the wage rate due to the FOC on the labor supply (22).[10] When the net total effect of extra savings on interest and wage rates is detrimental to the planner's resources, the externality of savings on factor prices is a cost for the planner and limits the increase in savings to value that means positive capital tax.

When the utility function is separable, the externality of savings on post-tax factor prices is proportional to the curvature of the utility function, and more precisely to the inverse of the IES. This can be easily seen with unemployed agents. Since their budget constraint is $R_{t+1}a_{e,t} = c_{u,t+1}$, the effect of a change in prices due to a marginal increase in savings (and thus higher consumption by unemployed agents) is proportional to $\frac{\partial R_{t+1}}{\partial c_{u,t+1}}a_{e,t}$, which is equal to $\frac{\partial \log R_{t+1}}{\partial \log c_{u,t+1}}$. Because of the Euler equation, this is the inverse of the intertemporal elasticity of substitution, σ_u. For employed agents, the extra savings (and hence the lower consumption of the employed) affect both the interest and the wage rates and yields the sum of elasticities $-a_{e,t}\frac{\partial \log R_{t+1}}{\partial c_{e,t}} - w_t l_{e,t}\frac{\partial \log w_t}{\partial c_{e,t}}$, which using the budget constraint (18), the Euler equation (20), and the FOC (22) on the labor supply becomes $-\sigma_e$. Therefore, in the separable case, the externality of savings on factor prices is proportional to the gap between the inverse IES of employed and unemployed agents. In the general non-separable case, the effect on the employed further includes a cross-derivative term, $\varsigma^l_{c,e}$ coming from the term $\frac{\partial \log w_t}{\partial c_{e,t}}$.

As a take-away, equation (30) states that when setting aggregate savings, the planner trades off the benefits of a larger capital tax base with the cost of higher post-tax factor prices. An alternative interpretation is that at the equilibrium, the planner uses the capital tax to correct the externality of savings on factor prices that would otherwise let the agents save "too much" from a social perspective: the planner uses their instruments to correct the negative externality of post-tax factor prices on savings.[11]

Additionally, the FOC related to the labor supply of the employed can be written as:

$$\frac{F_L - w}{w} = \Xi \times \left(\sigma_e + \frac{1}{\psi_e} - \varsigma^l_{c,e} + \varsigma^c_{l,e}\right), \tag{31}$$

[10]The effect actually depends on the cross derivative of the utility function with respect to consumption and labor. When the utility is separable, a lower consumption of employed agents always decreases the wage rate. When $U_{cl} < 0$, the effect is mitigated and can possibly have a different sign. For instance, with a GHH utility function, the wage rate is independent of employed consumption.

[11]This argument is reminiscent of the one in Dávila et al. (2012), where in the laissez-faire economy agents can save too much because they do internalize the effect of their savings on factor prices. In our environment, at the SRE the planner considers the effect of the price externality on the budget constraint of the government.

with the same multiplier $\Xi \geq 0$ as in FOC (30). The interpretation follows the same line as the interpretation of FOC (30). The planner trades off the benefit of an additional hour of labor supply in terms of a higher labor tax base, with the cost of higher post-tax factor prices. On the one hand, for the planner's resources (6), an additional hour of labor supply increases total output by $F_{L,t}$ at the cost of higher (after-tax) wage w_t. Thus, the effect is proportional to the gap between $F_{L,t}$ and w_t, and hence to the labor tax. An equivalent view is that the higher labor supply increases the base of the labor tax, and thus the planner's resources, in proportion to τ_t^L. On the other hand, the cost again operates through the externality of the labor supply on post-tax factor prices – and hence only concerns employed agents. Since the extra labor supply also increases consumption of the employed, the effect in the separable case is proportional to $\frac{\partial \log w_t}{\partial \log c_{e,t}} + \frac{\partial \log w_t}{\partial \log l_{e,t}}$, which equals $\sigma_e + 1/\varphi_e$. The non-separable case features two additional interaction terms, equal to $-\varsigma_{c,e}^l + \varsigma_{l,e}^c$ involving cross derivatives of the utility function.

Separable utility functions : CRRA, DRRA, CARA and IRRA cases. The equality (29) has important implications for specific utility functions. For separable utility functions, the cross-derivative terms are zero and $\varsigma_{c,e}^l = \varsigma_{l,e}^c = 0$, which simplifies the algebra (as well as the intuition, somewhat). We distinguish three cases: Constant, Decreasing and Increasing Relative Risk Aversion utility functions (CRRA, DRRA, IRRA respectively). Detailed calculations and numerical examples can be found in Appendix B.

First, the CRRA separable case, $U(c,l) = \frac{c^{1-\sigma}-1}{1-\sigma} - v(l)$, with $\sigma \neq 1$, or $U(c,l) = \log c - v(l)$, implies $\sigma_u = \sigma_e = \sigma$ independently of consumption levels. Since there is no externality of savings on prices, the planner sets savings so as to maximize their impacts on its resources: $1 + F_K = R$, or $\tau^K = 0$. Steady-state capital taxes are null in this equilibrium. This outcome generalizes the result of Chamley (1986) to an economy with an occasionally-binding credit constraint, but only for this specific utility function. It is consistent with the claims of Chen et al. (2020); Auclert et al. (2022); Chien and Wen (2023) – the latter provided a general proof considering the CRRA case. We summarize this result in the next corollary.

Corollary 1. *If the utility function is $U(c,l) = u(c) - v(l)$, where u is CRRA, then the capital tax is* 0 *in SRE.*

For other cases, observe that a positive capital tax implies $\beta R < 1$ and hence $c_u < c_e$ when credit constraints are binding. For DRRA functions, we have $\sigma_u > \sigma_e$ and the net distributional gains in (29) are positive. Savings imply a negative externality on post-tax factor prices. The planner avoids to increase these factor prices too much and ends up with a positive capital tax. Thus, the equilibrium, if it exists, will then feature positive capital and labor taxes. We provide examples of such equilibria for Stone-Geary utility functions in Appendix B.1.1 and for Fishburn utility functions in Appendix B.1.2.[12]

For the IRRA utility function, the situation is slightly more involved. On the one hand, savings still involve a negative externality on factor. However, since $\sigma_u < \sigma_e$, the negative externality of savings in (30) is the combination of savings *decreasing* interest rates and wages (rather than increasing them as in the DRRA case), and of post-tax factor prices having a *positive* externality on savings (and not negative as in the usual case discussed after equation (30)). The latter relationship implies that the labor supply in (30) has a positive externality on agents, and hence that labor should be subsidized. Therefore, the equilibrium, if it exists, features a positive capital tax but a *negative* labor tax. A standard example of an IRRA utility function is a Constant Absolute Risk Aversion (CARA) utility function, $U(c,l) = -\frac{1}{\gamma}e^{-\gamma c} - \frac{1}{\chi\varphi}e^{\varphi l}$, where $\gamma, \varphi > 0$. These functions are used by Acharya and Dogra (2021) and Acharya et al. (2023), among others. We have $\sigma(c) = \gamma c$, which is increasing. See Appendix B.2 for a numerical example with a CARA utility function.

Non-Separable utility functions: The GHH case. A standard non-separable utility function considered in the literature is the GHH utility function. This instantaneous utility function U is:

$$U(c,l) := u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right), \tag{32}$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ scales labor disutility, and the function u has a constant IES equal to $1/\sigma \geq 0$. This function has the property that the labor supply exhibits no wealth effect. It has been used for instance in the seminal contributions of Aiyagari (1995) and Diamond (1998) to obtain analytical results, but also in some quantitative work (e.g., Bayer et al., 2019 and Açikgöz et al., 2022), as it simplifies the computation of the equilibrium allocation.

Applying the equality (29) in the context of the GHH utility function yields the following relationships between smoothing and labor wedges:

$$1 - \beta R = \frac{F_L - w}{w}\varphi\sigma\left(1 + \beta\left(\beta R\right)^{\frac{1}{\sigma}-1}\right). \tag{33}$$

[12]Fishburn (1977) analyzes a utility function which is isoelastic below a threshold and linear after it. This utility function was used by Challe and Ragot (2016) and LeGrand and Ragot (2018) because it generates tractable models.

In the case where $\sigma = 1$ (and thus $u(c) := \log(c)$), this expression can be further simplified into a simple relationship relating the capital and labor taxes:

$$(1-\beta)\tau^K = \frac{\tau^L}{1-\tau^L}\varphi(1+\beta). \tag{34}$$

This shows that in equilibrium, the capital tax increases with the labor tax: both distortions increase together with the financial requirements that the planner has to finance.[13] In particular, the capital tax is positive whenever the labor tax is.[14]

Finally, for the sake of completeness, we also consider another example of a non-separable utility function, which is the one of King-Plosser-Rebelo (King et al., 1988), used for example by Dyrda and Pedroni (2022). We provide results in Appendix B.3. We find that an SRE with positive capital and labor taxes can exist only for some restrictions on the parameters, which is the case considered by Dyrda and Pedroni (2022).

3.2 An Existence Result in the GHH Case

We now focus on the GHH case with an IES of 1. We consider this utility function for the rest of the paper.

Before presenting these existence conditions, three remarks are in order. First, even in this simple framework, we must check that the Karush–Kuhn–Tucker conditions apply to our problem, and that the FOCs actually characterize an optimum. Because of the nonlinearity of the constraints (20)–(23), the standard Slater (1950) condition does not apply in our optimization program. Therefore, we must check another constraint qualification; this is done in Appendix C.2, where we verify that the linear independence constraint qualification holds. Second, we verify that the second-order conditions of the Ramsey planner are also fulfilled, such that the FOCs indeed characterize a maximum. This is done in Appendix C.3. Finally, we also consider an IES different from 1 in Appendix C.8, but we keep the simplest case in this Section.

3.2.1 First-best allocation and possible decentralization

As is standard in this type of problem, the first-best outcome can be attained if public spending is not too high. In this case, public debt is negative (the government thus holds

[13] One can check that τ^K/τ^L increases with the discount factor β and the Frisch elasticity.

[14] When β increases towards 1 from below, equation (34) would imply that the capital tax would increase without limit relative to the labor tax. However, the equilibrium does not exist in this case, as shown in Section 3.2 below, where the existence proof is provided.

a share of the capital stock) and the government finances public spending out of interest payments on its asset holdings. This is stated formally in the next proposition, whose proof can be found in Appendix C.1, together with the value of the steady-state first-best level of output Y_{FB}.[15]

Proposition 2. *Define:*

$$\overline{g}_1 := \frac{1-\beta}{\beta}\frac{\alpha}{1/\beta+\delta-1} - \frac{1-\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}. \tag{35}$$

If public spending verifies $G \leq \overline{g}_1 Y_{FB}$, then the steady-state Ramsey allocation is the first-best steady-state allocation characterized by zero taxes and perfect consumption smoothing.

When $G > \overline{g}_1 Y_{FB}$, the first-best allocation cannot be sustained, because financing such a large public spending out of capital income requires the government to hold a financial asset position that would exceed the total capital stock in the economy[16].

3.2.2 Binding credit constraints

When the first-best allocation cannot be sustained, the credit constraints of the unemployed agents must bind. The next proposition characterizes the existence of the equilibrium with credit constraints.

Proposition 3. *There exist two thresholds $\overline{g}_{La}$ and $\overline{g}_{SW}$, defined as:*

$$\overline{g}_{La} := \frac{1-\alpha}{\varphi}\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)(1-\overline{\tau}^L_{La})^{1+\varphi}, \tag{36}$$

$$\textit{where: } \overline{\tau}^L_{La} = \frac{1}{1+\varphi} - \frac{1}{1-\alpha}\frac{\varphi}{1+\varphi}\frac{\overline{g}_1}{1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}}, \tag{37}$$

$$\overline{g}_{SW} := \overline{g}_1 + (1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)(1-\frac{1}{1+\varphi(1+\beta)})^{\varphi}, \tag{38}$$

such that when $\overline{g}_1 Y_{FB} < G \leq \min(\overline{g}_{SW}, \overline{g}_{La}) \times Y_{FB}$, there exists a unique SRE with a binding credit constraint for unemployed agents, where both taxes τ^L and τ^K are positive.

The proposition is proved in Appendices C.4 and C.5. In addition to the non-first-best condition, $\overline{g}_1 Y_{FB} < G$, the existence of the steady-state equilibrium is subject to two

[15]This non first-best condition is the condition identified in more general settings by Albanesi and Armenter (2012), for the optimal steady-state capital tax not to be zero.

[16]This would imply that the government lends resources to households, which is prevented by credit constraints.

additional conditions, reflected in the two thresholds ($\overline{g}_{SW}$ and $\overline{g}_{La}$) for public spending. The first threshold $\overline{g}_{SW}$ ensures that the consumption of unemployed agents is positive and that the Lagrange multiplier μ on the governmental budget constraint (23) is constant and finite.[17] When G increases toward $\overline{g}_{SW}Y_{FB}$, the planner needs to raise capital taxes so high that the post-tax return of savings tends to zero, as does the consumption of unemployed agents. In this case, the government finds it infinitely costly to implement the steady-state optimal allocation, as taxes and distortions become infinitely high. This explains why the Lagrange multiplier on the governmental budget, μ, tends to infinity. At the threshold $\overline{g}_{SW}Y_{FB}$, the planner prefers to switch to a non-stationary equilibrium, where output is decreasing. This limit case has been discussed recently by Straub and Werning (2020), justifying the SW subscript and the denomination of Straub-Werning condition. To summarize, if $G > \overline{g}_{SW}Y_{FB}$, then no (stationary) steady-state equilibrium exists, and a non-stationary equilibrium may exist, as studied in Appendix C.6.[18]

The threshold $\overline{g}_{La}$ corresponds to a more traditional Laffer condition. When G is higher than this last threshold, not enough resources can be raised in the economy through the distorting taxes to finance public spending. We prove in Appendix C.4 that the constraints $\overline{g}_1 Y_{FB} < G \leq \min(\overline{g}_{SW}, \overline{g}_{La}) \times Y_{FB}$ are compatible for some G and some parameter values. However, stating which of $\overline{g}_{SW}$ or $\overline{g}_{La}$ is greater in general is not possible, as both cases are possible depending on parameter specification. For instance, when α is close to 1, we have $\overline{g}_{SW} < \overline{g}_{La}$, while when both α and φ are close to 0, the opposite holds.

The allocation can be derived explicitly in this tractable economy. Because of the GHH assumption, the labor supply of the employed agent is given by $l_e = (\chi w)^{\varphi}$, while their saving is $a_e = \frac{\beta}{1+\beta}\frac{w(\chi w)^{\varphi}}{1+\varphi}$. Using the expression (34) together with the government's budget constraint (23) at steady state, we can obtain an explicit expression for the post-tax real wage, thus providing an analytical solution to the allocation: $w = \frac{(F(k_{FB},1)-G)(1+\varphi)+w_{FB}\varphi}{1+2\varphi+\frac{1-\beta}{1+\beta}}$, where $w_{FB} = (1-\alpha)k_{FB}^{\alpha}$ and $k_{FB} = \left(\frac{\alpha}{\frac{1}{\beta}+\delta-1}\right)^{\frac{1}{1-\alpha}}$ are the first-best capital-to-labor ratio and wage rate, respectively (see Appendix C for further details). The labor tax is then $\tau^L =$

[17] In the case of the log-GHH utility function, the two conditions are identical. When the IES differs from one, the conditions $c_u > 0$ and $\mu > 0$ differ from each other, and the condition $\mu > 0$ typically binds first. See Appendix C.8 for further details. In addition we also show that the allocation and the dynamic of Lagrange multipliers are continuous in the IES, such that the specific case identified by Lansing (1999) and Reinhorn (2019) (i.e. diverging multipliers and converging allocation) for the IES of 1 does not exist in our environment.

[18] When β increases from below towards 1, we find that the Straub-Werning threshold decreases and $\overline{g}_{SW} < \overline{g}_1$, implying that there is no steady-state equilibrium for any G (see equations (38) and (96) in the Appendix). As a consequence, the capital tax cannot increase without bound as equation (34) would imply.

$1-w/w_{FB}$ and the capital tax is given by equation (34). Finally, public debt can be deduced from the capital market clearing condition: $B = a_e - K = (\chi w)^{\varphi}(\frac{\beta}{1+\beta}\frac{1-\tau^L}{1+\varphi}w_{FB} - k_{FB})$.[19]

3.2.3 When is optimal public debt positive?

This model can generate a positive amount of optimal public debt, as stated in the following result, which is proved in Appendix C.7.

Result 1. *There exists a threshold $\overline{g}_{pos}$ defined as:*

$$\overline{g}_{pos} = \frac{1+\beta}{1-\beta}(1+2\varphi)(-\overline{g}_1), \tag{39}$$

such that steady-state public debt is positive, $B \geq 0$, iff $\overline{g}_1 \leq 0$ and $G \leq \overline{g}_{pos} Y_{FB}$.

The joint positivity of public debt and capital tax is not obvious: why would the planner provide more public debt to the market (more liquidity in the sense of Woodford, 1990) and then tax the return on public debt with a positive capital tax? In an equilibrium with positive public debt, the equilibrium savings of employed agents are higher than the optimal capital stock, and the extra savings are absorbed by the public debt. From this allocation, decreasing public debt would inefficiently increase the capital stock, and would further require an increase in the capital tax to reduce savings, which would hinder consumption smoothing. Thus, public debt enables the planner to absorb the excess of savings and reconcile the high savings of private agents with the optimal capital stock without affecting consumption smoothing too drastically. This explains the condition $\overline{g}_1 \leq 0$, which states that, no matter the level of public spending, it is never optimal for the government to hold a share of capital to finance public spending. The agents' savings motives are indeed too strong given the level of capital in the economy. The second condition $G \leq \overline{g}_{\text{pos}} Y_{FB}$ comes from the fact that a high level of public spending requires a high level of distorting taxes, and thus a lower level of private saving. As a consequence, the public debt necessary to absorb the excess saving is decreasing with G. If G is too high, optimal debt becomes negative.

[19] When an SRE with a positive capital tax exists, we can show that the planner does not want to implement any lump-sum transfers, since they would require raising distortionary taxes to implement. The proof is in Appendix C.4.

3.2.4 Conclusion about existence

We have characterized four conditions, given by $\overline{g}_1, \overline{g}_{SW}, \overline{g}_{La}$ and $\overline{g}_{\text{pos}}$ for the existence of an SRE with positive optimal capital taxation and public debt. These conditions are independent, but can be satisfied simultaneously for some parameter values.[20] For instance, we use the following parameters (ensuring existence) to study the steady state and dynamics of the simple model in Appendix D.3. The parameters are $\alpha = 0.3, \beta = 0.7, \varphi = 0.3, \delta = 1, G = 0.01, \chi = 1$, and one can check that $\overline{g}_1 Y_{FB} < G$, $G \leq \overline{g}_{SW} Y_{FB}$, $G < \overline{g}_{La} Y_{FB}$, and $G < \overline{g}_{pos} Y_{FB}$. This economy has an equilibrium capital tax of 6%, a labor tax of 3%, and a (small) positive public debt. Larger values of public spending G reduce the public debt, which can become negative. There is actually no interior steady state, as discussed in Appendix C.5, although non-interior equilibria may exist. Since these properties are close to those derived by Straub and Werning (2020), we present them in Appendix C.6.

3.3 Dynamic Analysis of Public Debt

We now use the simple model to derive some insights about the optimal dynamics of public debt after a public spending shock. We assume full capital depreciation, $\delta = 1$, and consider a first-order approximation of the model.

Time consistency. It is interesting to note that in the log-GHH case with log period utility (32), the program of the planner is time-consistent, although capital is fixed in period 0 and capital taxes are chosen at period 0 (which is not the case in more general settings, as discussed below and in LeGrand and Ragot, 2023). Indeed, in this case, and when credit constraints bind, the saving of employed agents does not depend on the post-tax real interest rate, but only on the post-tax real wage (see Appendix C.4).[21]

Linearization. We denote with a hat the relative deviation of a variable from its steady-state value: $\widehat{x}_t = \frac{x_t - x}{x}$ for a generic variable x_t with steady-state value x. The public

[20] Regarding uniqueness, in the log-GHH case, we can prove the uniqueness of the SRE. However, this is not true in the general case, when the IES is different from one. Even with a GHH utility function, if the IES differs from one, multiple allocations can satisfy the planner's FOCs while satisfying the Straub-Werning and Laffer conditions. See Appendix C.8 for a numerical example.

[21] More precisely, the Lagrange multipliers on the previous period's Euler equations do not affect the current period allocation. See Appendix C.4.

spending shock is assumed to be defined as follows:

$$\widehat{G}_t = \begin{cases} \widehat{G}_0 & \text{if } t = 0, \\ \rho_G \widehat{G}_{t-1} & \text{if } t > 0, \end{cases} \tag{40}$$

with $\widehat{G}_0$ small enough for a first-order approximation of the dynamics to be relevant, and $\rho_G \in (-1, 1)$. The shock only happens at date $t = 0$ and then persists with parameter ρ_G, as is consistent with our assumption of an MIT shock.

Characterization of the system stability. Our first result is to characterize the stability of the dynamic system that yields the Ramsey allocation, using the FOCs of the planner. Interestingly, the dynamic of the Ramsey allocation can be fully summarized by taking capital as the unique state variable, together with the public spending shock.

Result 2. *The optimal dynamic of the capital stock is given by the following system:*

$$\widehat{K}_t = \rho_K \widehat{K}_{t-1} + \sigma_K \widehat{G}_t, \tag{41}$$

where the coefficients $\rho_K > 0, \sigma_K < 0$. ρ_K *does not depend on* ρ_G *and* $\frac{\partial \sigma_K}{\partial \rho_G} > 0$.

See Appendix D.1 for the expressions of the coefficients and computations. Thus at impact, an increase in public spending diminishes capital, and the higher the persistence of the public spending shock, the stronger the effect.

The dynamic system (41) is stable when the auto-regressive coefficient ρ_K is smaller than one in absolute value. In our setup, this is equivalent to verifying the Blanchard–Kahn conditions. The result regarding system stability is summarized in the following proposition.

Proposition 4. *The system (41) is stable, i.e.,* $|\rho_K| < 1$, *iff*

$$\alpha \leq \frac{1}{1 + (1 - \beta)(1 + \varphi)}. \tag{42}$$

The dynamic system is stable under (42), which is called the Blanchard-Kahn condition and which imposes an upper bound on α. Note that this upper bound is always strictly smaller than one and hence can be binding. Condition (42) on α always holds when public debt is positive, i.e., when $\overline{g}_1 < 0$. When the capital share α and the Frisch elasticity φ are both high (such that (42) is not fulfilled), a small shock in public spending induces too large a change in the resources of the planner, such that capital diverges.

By induction, we can derive from (40) and (41) the closed-form expression of the impulse response function for optimal capital:

$$\widehat{K_t} = \sigma_K \widehat{G}_0 \frac{\rho_K^{t+1} - \rho_G^{t+1}}{\rho_K - \rho_G}. \tag{43}$$

This allows us to completely characterize the capital path following a public spending shock. At impact and after a positive shock ($\widehat{G}_0 > 0$), the relative variation of capital is always negative by a quantity $\sigma_K \widehat{G}_0 < 0$. Then, the profile of the capital variation is hump-shaped: it starts decreasing further, before increasing and reverting back to zero (see Appendix D.2 for further characterization of the dynamics of the capital stock).

Role of the persistence of the public spending shock ρ_G on public debt. From the expression for capital (43), it is possible to derive an explicit expression for the optimal dynamics of public debt:

$$\widehat{B}_t = \widehat{G}_0(\Theta^K \rho_K^t - \Theta^G \rho_G^t). \tag{44}$$

The coefficients Θ^K, Θ^G are functions of the parameters of the model but not of $\widehat{G}_0$ and are provided in equations (127) and (128) of Appendix D.2. These parameters can be either positive or negative. As a consequence, on impact, the change in public debt, $\widehat{B}_0 = \widehat{G}_0(\Theta^K - \Theta^G)$, after a positive public spending shock ($\widehat{G}_0 > 0$) can be either positive or negative, because the sign $\Theta^K - \Theta^G$ is ambiguous. We can characterize the effect of the persistence of the shock on the initial change of public debt, considering two cases. First, we analyze the effect of ρ_G with fixed $\widehat{G}_0$ to understand the mechanisms. Our second experiment focuses on studying the effect of ρ_G while keeping the NPV of public spending unchanged. More formally, we keep the following quantity unchanged, denoted by $N\hat{P}V_0$:

$$N\hat{P}V_0 = \sum_{t=0}^{\infty} \frac{\hat{G}_t}{R^t} = \widehat{G}_0 \sum_{t=0}^{\infty} \left(\frac{\rho_G}{R}\right)^t = \widehat{G}_0 \frac{R}{R - \rho_G}.$$

Keeping the NPV unchanged while changing ρ_G implies setting the initial size of the shock to $\widehat{G}_0(\rho_G) = N\hat{P}V_0 \frac{R-\rho_G}{R}$. This is summarized in the following proposition.

Proposition 5. *Assume that the steady-state public debt is positive: $B > 0$. Denoting by $\widehat{B}_0$ the variation of public debt on impact, we have*

$$\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{\widehat{G}_0} < 0.$$

Moreover, if we further assume $\widehat{B}_0 > 0$, we also have

$$\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{N\hat{P}V_0} < 0.$$

See Appendix D.2 for the proof. The intuition for why the dynamic of the debt depends on the persistence of the shock is the following. After a positive public spending shock, capital is always falling, but to implement consumption smoothing the planner does not want to decrease private savings (which are used by unemployed agents to consume). Consequently, when the persistence of the shock is low, the planner increases public debt to provide a store of value to private agents. Then, a small increase in future taxes allows one to reduce public debt. When the persistence is high, this strategy is very costly in terms of welfare, because the fall of the capital stock is persistent, and the planner would have to increase taxes to reduce public debt in periods when capital and output are low. Consequently the planner does not increase public debt, in order to avoid having to raise taxes in the future to stabilize this debt. Finally, we check in Appendix D.3 with a simple numerical example that Proposition 5 still holds when we consider a non-marginal variation in the persistence.

3.4 Ex-Ante Heterogeneous Populations and Social Weights

We extend the previous GHH case, introducing further heterogeneity between agents, in order to analyze the role of social weights in the determination of the optimal capital tax. We now consider two types of agents facing different labor income risk ($F = 2$ using the notation from Section 2). We consider an environment where one type of agent always remains employed, and will not save in equilibrium (reproducing the environment of Judd, 1985), whereas the other type of agent alternates between employment and unemployment, similarly to the agent in Section 3.1. We use the superscript A to denote the type that alternates between employment and unemployment, and the superscript B for the agents who remain employed, and who will not save in equilibrium. The productivity of employed agents is denoted y^A and y^B. The population size of each type is denoted by $\Omega^x \in [0, 1]$, $x = A, B$ with $\Omega^A + \Omega^B = 1$. The planner deviates from a Utilitarian objective, and the social welfare weights on the two types of agents (may) differ from their actual shares in the population. The weights in the social welfare function are denoted by $\omega^x \in [0, 1]$, $x = A, B$ with $\omega^A + \omega^B = 1$.

The market clearing condition implies $L_t = \Omega^A y^A l^A_{e,t} + \Omega^B y^B l^B_{e,t}$ for the labor market and $A_t = \Omega^A a^A_{e,t}$ for the capital market. The planner's objective can be written as:

$$\omega^A \sum_{t=0}^{\infty} \beta^t \left(\log(c^A_{e,t} - \chi^{-1} \frac{l_{e,t}^{A,1+1/\varphi}}{1+1/\varphi}) + \log(c^A_{u,t}) \right) + \omega^B \sum_{t=0}^{\infty} \beta^t \log(c^B_{e,t} - \chi^{-1} \frac{l_{e,t}^{B,1+1/\varphi}}{1+1/\varphi}). \quad (45)$$

where we restrict to the log case as before. For the sake of simplicity, we define:

$$\Lambda := \frac{\Omega^B w y^B l^B_e}{\Omega^A w y^A l^A_e} = \frac{\Omega^B (y^B)^{\varphi+1}}{\Omega^A (y^A)^{\varphi+1}},$$

which is the ratio between labor income of type B agents and that of type A agents. It captures the inequality in labor income between the two populations. The further away Λ is from 1, the greater the inequality. The following proposition summarizes our main result.

Proposition 6. *In any interior SRE, the smoothing and labor wedges verify:*

$$\underbrace{\frac{1-\beta R}{\omega^A}}_{\text{Smoothing wedge}} = \underbrace{\frac{\omega^B}{\omega^A} - (1+\beta)\Lambda}_{\text{Redistribution}} + \underbrace{\frac{F_L - w}{w}}_{\text{Labor wedge}} \varphi \underbrace{(1+\beta)(1+\Lambda)}_{\text{Distributional Gain}}, \quad (46)$$

or equivalently, capital and labor taxes are related by

$$(1-\beta)\frac{\tau^K}{\omega^A} = \frac{\omega^B}{\omega^A} - (1+\beta)\Lambda + \varphi(1+\beta)(1+\Lambda)\frac{\tau^L}{1-\tau^L}. \quad (47)$$

The proof can be found in Appendix E. Proposition 6 shows that social preferences (ω^A, ω^B) and the optimal tax system are intertwined. On the one hand, from a given set of social preferences (ω^A, ω^B), the fiscal policy (τ^K, τ^L, B) and the SRE can be derived. In fact, equation (47) gives τ^K as function of τ^L, which is then pinned down by the government budget constraint. Public debt is given by the financial market clearing condition. On the other hand, for a given fiscal policy (τ^K, τ^L, B) and an SRE equilibrium, the social preferences (ω^A, ω^B) of the planner can be derived from equation (47). The latter relationship is known as the *inverse optimal approach*, where the social weights are found in order to replicate an observed fiscal system, which is assumed to be optimal. This approach is used in the general model of Section 4, where we characterize the weights that allow the model's steady state to replicate the actual US fiscal system, in order to study its dynamics.

What is the intuition of equation (46)? Since only type-A agents save, while both types supply labor, the distortions associated with the interest rate affect only type-A agents while both types are affected by the distortions associated with the wage rate. This explains why the smoothing wedge is divided by the weight of type-A agents, ω^A, while the labor wedge is actually divided by the weight of the total population $\omega^A + \omega^B = 1$. The relationship (46) includes a third term called "redistribution", which comes from the fact that social weights of agents differ from their *no-distribution weights*. These latter weights are equal to the inverse of marginal utility, such that the planner would not want to implement any redistribution between types. In our log-GHH setup, $(1+\beta)\Lambda$ is equal to the ratio of type-A marginal utilities to type-B marginal utilities, and thus to the ratio of no-distribution weights. The further the social weights are from the no-distribution weights, the higher the capital tax is relative to the labor tax. This is consistent with the fact that type-B agents pay only the labor tax (but not the capital tax), so increasing their social weight increases the social welfare impact of higher labor taxes. The effect of the weight on the redistribution term always dominates its effect on the smoothing wedge term. Thus, social weights play an intuitive role in the composition of the tax scheme: A higher weight ω^B on type B agents increases the capital tax relative to the labor tax. Finally, we can observe that the effect of a higher actual share Ω^B of type B agents on the capital tax (i.e., a higher Λ) is ambiguous, as can be seen in equation (47). On the one hand, it increases the total labor income in the economy (since B-agents are always employed), which increases the tax base of the labor tax, allowing a reduction of the labor tax (for a given capital tax). On the other hand, a higher share Ω^B reduces the redistribution motive as it reduces the gap between relative social weights and relative no-distribution weights. This tends to raise the labor tax relative to the capital tax.

4 The General Model

We now show that the previous results about the price externality and the optimal public debt dynamics hold in the general model. We now analyze the model of Section 2, while considering a GHH utility function.

4.1 Description and planner's FOCs

Taking advantage of the GHH utility function allows us to simplify the Ramsey program, with some changes of variables. First, the labor choice of an agent i of type f, or an (i,f)-agent in short, given by (11) can be written as $l_{i,t}^f = l_t (y_{i,t}^f)^{\frac{1-\tau_t}{1/\varphi+\tau_t}}$, where:

$$l_t := (\chi(1-\tau_t)w_t)^{\frac{1}{1/\varphi+\tau_t}}. \tag{48}$$

The quantity l_t can be interpreted as the labor supply of an agent endowed with a productivity of 1, and will hence be called the *unitary labor supply*.[22] Second, we define an increasing transformation of the progressivity τ_t as

$$\tilde{\tau}_t := \frac{(1/\varphi + 1)(1 - \tau_t)}{1/\varphi + \tau_t} \in (0, 1 + \varphi], \tag{49}$$

where $\tilde{\tau}_t = 1 + \varphi$ corresponds to linear taxation and $\tilde{\tau}_t \to 0$ to full income redistribution (i.e., $\tau_t = 1$, which is always a dominated option for the planner). Third, we define the aggregate quantity $x^f_{i,t} := c^f_{i,t} - \chi^{-1}\frac{(l^f_{i,t})^{1+1/\varphi}}{1+1/\varphi}$, such that the period utility of an agent (i, f) is simply denoted $u(x^f_{i,t})$, while their budget constraint is:

$$x^f_{i,t} = (1 + r_t)a^f_{i,t-1} - a^f_{i,t} + \frac{1}{\chi\tilde{\tau}_t} l_t^{1/\varphi+1}(y^f_{i,t})^{\tilde{\tau}_t}.$$

We can similarly rewrite the governmental budget constraint (6) using this notation. Formally, the Ramsey program can be written as follows:[23]

$$\max_{\left(r_t,\tilde{\tau}_t,B_t,K_t,L_t,l_t,((a^f_{i,t},x^f_{i,t},\nu^f_{i,t})_{i\in\mathcal{I}^f})_{f\in\{1,\ldots,F\}}\right)_{t\geq 0}} \sum_{f=1}^{F} m^f \omega^f \sum_{t=0}^{\infty} \beta^t \int_i u(x^f_{i,t})\ell^f(di), \tag{50}$$

$$G_t + T_t + r_t A_{t-1} + \left(\frac{1}{\tilde{\tau}_t} + \frac{1}{1/\varphi + 1}\right)\frac{l_t^{1/\varphi+1}}{\chi}\sum_{f=1}^{F} m^f \int_i (y^f_{i,t})^{\tilde{\tau}_t}\ell(di) = F(A_{t-1} - B_{t-1}, L_t) + B_t - B_{t-1} \tag{51}$$

$$\text{for all } i, f:\ x^f_{i,t} = (1 + r_t)a^f_{i,t-1} - a^f_{i,t} + \frac{1}{\chi\tilde{\tau}_t} l_t^{1/\varphi+1}(y^f_{i,t})^{\tilde{\tau}_t}, \tag{52}$$

$$u'(x^f_{i,t}) = \beta\mathbb{E}_t[(1 + r_{t+1})u'(x^f_{i,t+1})] + \nu^f_{i,t}, \tag{53}$$

$$a^f_{i,t} \geq -\bar{a},\ \nu^f_{i,t}(a^f_{i,t} + \bar{a}) = 0,\ \nu^f_{i,t} \geq 0,\ x^f_{i,t} \geq 0,\ l^f_{i,t} \geq 0, \tag{54}$$

$$A_t = \sum_{f=1}^{F} m^f \int_i a^f_{i,t}\ell^f(di),\ L_t = l_t \sum_{f=1}^{F} m^f \int_i (y^f_{i,t})^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}}\ell^f(di). \tag{55}$$

Once the previous program has been solved, we can recover τ_t, w_t, $l^f_{i,t}$ and $c^f_{i,t}$ from the resulting allocation. The constraints guarantee that the governmental budget is balanced in (51) and that the planner actually selects a competitive equilibrium characterized by individual budget constraints (52), individual Euler equations (53), individual credit and positivity constraints (54), and market clearing conditions (55).

In Section 3, we derived the planner's FOCs using a primal approach, where prices

[22] This change in variable makes it unnecessary to solve for the labor supply of individual agents.

[23] The condition $x^f_{i,t} \geq 0$ and $l^f_{i,t} \geq 0$ imply $c^f_{i,t} > 0$.

are substituted in using the FOCs of households (e.g., as in Bhandari et al., 2021). Here, we use the factorization approach, based on Marcet and Marimon (2019) and developed in LeGrand and Ragot (2022a). Both methods provide the same FOCs, as we show in Section F.2. However, the factorization approach is better suited to the interpretation and the resolution of the general case. The goal of Marcet and Marimon (2019) is to provide a recursive formulation for optimization problems with forward looking constraints (which here are the Euler equations of unconstrained agents). At the beginning of their construction (see equations (5) and (6) of Marcet and Marimon, 2019), they show that one can write the Lagrangian and then manipulate the terms to maximize the discounted sum of a single term. This term embeds forward-looking constraints and has no expectation term. This is the first step before writing a recursive formulation. We do not use the recursive formulation in our paper and only derive FOCs of the sequential problem. We thus avoid the question of the existence of a Bellman equation for the planner, where the Lagrange multipliers on the Euler equations would be state variables.

We denote as $\beta^t \lambda_{i,t}^f$ the Lagrange multiplier on the period-t Euler equation (53) of agent i of type f. When the credit constraint of agent i is binding, we have $a_{i,t}^f = -\bar{a}$ and $\lambda_{i,t}^f = 0$ because the Euler equation is not a constraint. When the credit constraint does not bind, the equilibrium can feature either $\lambda_{i,t}^f > 0$ or $\lambda_{i,t}^f < 0$ depending on whether the agents save too much or too little (from the planner's perspective). Similarly, we denote by $\beta^t \mu_t$ the Lagrange multiplier on the government budget constraint (51).

To save space, we derive the planner's FOCs in Appendix F, and provide the main results here. Note that we follow the literature and assume that the solution is interior and that the planner's FOCs are sufficient to characterize the optimal allocation. We provide some quantitative checks below.

To simplify the interpretation of the FOCs of the Ramsey program, we introduce the marginal social valuation of liquidity for agent i of type f defined as:

$$\psi_{i,t}^f := \omega^f u'(x_{i,t}^f) - (\lambda_{i,t}^f - (1 + r_t)\lambda_{i,t-1}^f) u''(x_{i,t}^f). \tag{56}$$

This complex expression has a simple interpretation. It is the value for the planner of transferring one unit of resources to agent i of type f (if possible). First, the extra unit is valued by the marginal utility weighted with the proper weight, $\omega^f u'(x_{i,t}^f)$. Second, this extra unit of resources also affects the savings incentives, both from period $t-1$ to t (the term in $\lambda_{i,t-1}^f$) and from period t to $t+1$ (the term in $\lambda_{i,t}^f$). These last two effects are weighted by the variation in marginal utility of consumption, $u''(x_{i,t}^f)$.

From equation (56), we also define the marginal value of the public funds financed by agent (i, f):

$$\hat{\psi}_{i,t}^{f} := \mu_t - \psi_{i,t}^{f} \tag{57}$$

This is the net value for the planner of transferring one unit of resources to its budget from an agent (i, f) .With this notation, the FOCs of the planner are easily interpreted. First, for an unconstrained agent (i, f), the planner implements a public-funds smoothing condition:

$$\hat{\psi}_{i,t}^{f} = \beta \mathbb{E}_t[(1 + r_{t+1})\hat{\psi}_{i,t+1}^{f}], \tag{58}$$

where because of the assumption of MIT shocks, the expectation is taken with respect to the idiosyncratic risk. Equation (58) is a generalized version of the Euler equation (10) (and is actually the same equation when all Lagrange multipliers are zero and all weights are set to 1), in which the planner internalizes through $\hat{\psi}_{i,t}^{f}$ the general equilibrium externalities when setting individual savings. For credit-constrained agents, we have $\lambda_{i,t}^{f} = 0$, and the Euler equation is not a constraint.

Here we present FOCs related to the fiscal tools. The FOC with respect to public debt can be written as

$$\mu_t = \beta(1 + \tilde{r}_{t+1})\mu_{t+1} \tag{59}$$

without an expectation operator because of the MIT shock assumption. Equation (59) shows that the planner aims at smoothing the shadow cost of the government budget constraint through time. This yields the modified golden rule at the steady state, as previously discussed.

The other FOC with respect to the post-tax interest rate captures the effect of a change in the capital tax:

$$\underbrace{\sum_{f=1}^{F} m^f \int_i \hat{\psi}_{i,t}^{f} a_{i,t-1}^{f} \ell^f(di)}_{\text{Net distributive gain}} = \underbrace{\sum_{f=1}^{F} m^f \int_i \lambda_{i,t-1}^{f} u'(x_{i,t}^{f}) \ell^f(di).}_{\text{Cost on savings incentives}} \tag{60}$$

A change in the capital tax generates benefits for the government through the taxation of heterogeneous households. Because the capital tax is levied on agents' asset holdings, the benefits are proportional to their beginning-of-period wealth, which is the net distributive effect (which is the term at the left-hand side (LHS)). These benefits are set equal to the costs, which operate through the savings incentives. From the planner's perspective, these costs depend on the Lagrange multiplier $\lambda_{i,t-1}^{f}$ on the Euler equation of each agent (term at the right-hand side (RHS)).

The FOC capturing the effect of a change in the labor supply l_t is:

$$\underbrace{\frac{1+1/\varphi}{\chi\tilde{\tau}_t} l_t^{1/\varphi+1} \sum_{f=1}^{F} m^f \int_i \hat{\psi}_{i,t}^f (y_{i,t}^f)^{\tilde{\tau}_t} \ell(di) \ell^f(di)}_{\text{Net welfare effect}} = \tag{61}$$

$$\underbrace{\mu_t \left(\sum_{f=1}^{F} m^f \int_i \left(\frac{l_t^{1/\varphi+1}}{\chi} (y_{i,t}^f)^{\tilde{\tau}_t} - (y_{i,t}^f)^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}} F_{L,t} l_t \right) \ell^f(di) \right)}_{\text{Reduction in goverment income}}. \tag{62}$$

As in FOC (60), the benefit of setting the labor tax level consists in public-funds transfers weighted by the tax base, which here is the labor supply, equal to $\frac{1}{\chi\tilde{\tau}_t} l_t^{1/\varphi+1} (y_{i,t}^f)^{\tilde{\tau}_t}$, for each agent (LHS). The cost is related to the modification of labor supply incentives that are affected by labor tax (RHS).

The FOC for the progressivity coefficient $\tilde{\tau}_t$ has a similar interpretation:

$$0 = \underbrace{\frac{l_t^{1+1/\varphi}}{\chi\tilde{\tau}_t} \sum_{f=1}^{F} m^f \int_i \hat{\psi}_{i,t}^f (y_{i,t}^f)^{\tilde{\tau}_t} \left(-\frac{1}{\tilde{\tau}_t} + \log y_{j,t}^f \right) \ell(di)}_{\text{Net distributive gain}} \tag{63}$$

$$\underbrace{- \mu_t \frac{l_t}{1/\varphi+1} \left(\sum_{f=1}^{F} m^f \int_i \log y_{j,t}^f \left(\frac{l_t^{1/\varphi}}{\chi} (y_{i,t}^f)^{\tilde{\tau}_t} - (y_{i,t}^f)^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}} F_{L,t} \right) \ell(di) \right)}_{\text{Cost on labor supply incentives}}.$$

Setting the progressivity of the labor tax is very similar to setting its level. Indeed, on the one hand, benefits are public-funds transfers weighted by the tax base. On the other hand, the costs are related to the modification of labor supply incentives. However, even though setting the average tax level or the progressivity (coefficient τ_t) has similar effects, they are two independent instruments because they affect the distribution of agents differently.

4.2 Inverse Optimal Approach at the Steady State

A quantitative simulation of the model requires taking a stand on the SWF, which is determined by the social weights $(\omega^f)_{f=1,\ldots,F}$. Indeed, our goal is to study the dynamics around a quantitatively relevant steady-state fiscal system. This is typically not the case when the steady state corresponds to a utilitarian planner with a standard calibration. To overcome this difficulty, we implement an inverse optimal approach (Bourguignon and Amadeo, 2015; Chang et al., 2018; Heathcote and Tsujiyama, 2021), allowing us to estimate the weights of the SWF. The inverse optimal approach consist in identifying the objective of the planner, for which an observed competitive equilibrium (i.e., observed allocation and instrument values) is socially optimal. More precisely, we calibrate the parameters of the model to obtain a realistic steady-state allocation given fiscal parameters set to match the actual US fiscal policy. Second, we find the values of the social welfare weights $(\omega^f)_{f=1,\dots,F}$, such that the chosen fiscal parameters are actually optimal for the planner at the steady state, and are a solution of an SRE. Once the social weights have been obtained, we can use this steady-state allocation to implement public spending shocks (with different persistences) to observe the responses of fiscal instruments. As these shocks are transitory, we can check that the value of the fiscal tools return to their initial values, which are the optimal ones in the long run. Overall, the FOCs of the planner are used twice: (i) at the steady state, to estimate the weights of the SWF from the actual fiscal system; (ii) in the dynamics to compute the optimal response of instruments given the estimated weights.

How many SWF weights can be identified from the steady-state FOCs of the planner? Fiscal policy is composed of four instruments $(\tau^K, B, \kappa, \tau)$, but these four instruments actually impose only two constraints on social weights. Indeed, fiscal policy is constrained by the budget constraint of the government, which removes one degree of freedom. Moreover, the public debt FOC (59) imposes a steady-state value on the before-tax real interest rate $1+\tilde{r} = 1/\beta$, but does not restrict the social weights. As the social weights are unique up to an increasing transformation, we further impose without loss of generality that the weights sum up to 1: $\sum_{f=1}^{F} m^f \omega^f = 1$. Given this normalization and the two FOC constraints, $F = 3$ different types of agents are needed to exactly identify the SWF weights from the FOCs of the planner. We will thus consider $F = 3$ in our quantitative exercise of Section 5.[24]

4.3 Consistency with the Analytical Model

After constructing the SRE using the inverse optimal approach, it can be checked that the conditions identified in the discussion of Proposition 2 are satisfied in the general model. First, the social weights ω^f are constructed so that the planner's FOCs hold. In this SRE we can check that the Lagrange multiplier of the government's budget is positive and that the Straub-Werning condition holds. Second, since the government's budget constraint holds with an interior fiscal policy, the Laffer condition is satisfied. Third, we also verify that the total return on the capital is not sufficiently large to finance public

[24] Considering $F > 3$ types is possible, at the cost of additional restrictions, such a the minimal deviation to the Utilitarian SWF. This is done in Section 5.4 below.

spending, and therefore that a first-best equilibrium cannot exist for the calibration under consideration. These three conditions correspond to those listed in Proposition 2 about the equilibrium. Furthermore, since the calibration implies a positive public debt, the condition in Proposition 5 also holds. Finally, when we simulate the dynamics of the model, we check that the Blanchard-Kahn condition holds for the truncated model, similarly to the condition in Proposition 4. These checks make us confident that we consider the perturbation of a relevant SRE.

Identifying the price externality. A result of the analytical model, and of Proposition 1, is that externalities of savings and labor choices on prices are key to pin down the optimal fiscal system. In the resolution of the general model (see Section 4.1), we rely on the Lagrangian approach to compute the planner's FOCs. We verify in Appendix F.2.1 that the FOCs derived with the Lagrangian or the primal approaches are identical, even though the Ramsey problems are solved differently.[25] With this approach, the price externality is captured by the Lagrange multipliers $\lambda_{i,t}$ indicating whether agent i is saving too much or too little from a social perspective. In the absence of price externalities, agents' private savings decisions would also be socially optimal and $\lambda_{i,t} = 0$ for all agents i (i.e., the Euler equation would not be a constraint for the planner but a redundant optimality condition). As in the analytical model, the absence of price externalities implies a zero optimal capital tax. Indeed, when $\lambda_{i,t} = 0$ for all agents i, equation (56) then implies that $\psi_{i,t}^f = \omega^f u'(x_{i,t}^f)$, which, for unconstrained agents, yields with equation (58):

$$\mu_t - \omega^f u'(x_{i,t}^f) = \beta \mathbb{E}_t R_{t+1} \left(\mu_{t+1} - \omega^f u'(x_{i,t+1}^f) \right).$$

This simplifies into $\mu_t = \beta R_{t+1} \mu_{t+1}$ using the Euler equation of agent i and the MIT shock assumption. Equation (59) then implies that pre- and post-tax rates are equal to each other: $1 + \tilde{r}_t = R_t$, which means a zero-capital tax: $\tau_t^K = 0$. Obviously, no stationary equilibrium would exist in that case. Indeed, savings would diverge and marginal utilities would tend to 0, implying a non-stationary equilibrium (see Chamberlain and Wilson, 2000).

Furthermore, we can prove that a positive capital tax comes with binding credit constraints at the steady state, as is the case in the simple model. Indeed, integrating the

[25] Additionally, in Appendix F.2.2, we check that the solution of the analytical approach is quantitatively similar to the limit of the solution of the general approach when the transition matrix converges to the anti-diagonal matrix of Assumption A (see Figure 5 in Appendix).

Euler equations (53) of all agents yields, at the steady state:

$$\tau^K = \frac{\sum_{f=1}^F m^f \int_i \nu_i^f \ell^f(di)}{(1-\beta)\sum_{f=1}^F m^f \int_i u'(x_i^f)\ell^f(di)}, \tag{64}$$

where $\nu_i^f \geq 0$ is the Lagrange multiplier on an individual's credit constraint. Equation (64) shows that $\tau^K > 0$ when a positive mass of agents having face a binding credit constraint at the steady state. As a consequence, having credit constraints that occasionally bind is a necessary condition for a positive optimal capital tax in an SRE.

4.4 Time-Inconsistency: Time-0 and Timeless Perspectives

In this general model, optimal policies are time inconsistent. This can be seen in the planner's FOCs, where past values of Lagrange multipliers $\lambda_{i,t-1}^f$ appear (see equations (56) used in (58)). In period 0, these Lagrange multipliers are typically initialized to zero. This means that in period 0, even in the absence of any shock, the planner is not bound by any past commitments. Obviously, this differs from the steady state, where past commitments matter (and past Lagrange multiplier values differ from zero). Thus, the planner has different incentives in period 0 than in the steady state and therefore deviates from the steady-state allocation. This is called a reoptimization shock and involves the time inconsistency of the planner's program in period 0.[26] Since we do not want our results to be affected by this effect, we neutralize the time inconsistency by setting the values of the Lagrange multipliers in period -1 to their steady-state values. This means that the planner faces the same commitments as in the steady state and it removes their incentives to deviate. In this case, in the absence of a shock, the economy optimally remains at its steady-state equilibrium.[27]

4.5 Numerical Tools

We solve the model using the tools of LeGrand and Ragot (2022a). This so-called truncation method generates a large but finite state space, allowing one to easily estimate the weights

[26] We study this time inconsistency and the reoptimization shock in LeGrand and Ragot (2023). A reoptimization shock (i.e., setting the Lagrange multipliers to 0 in period 0) generates a transitory dynamics even in the absence of external shock. In the case of the first-order perturbation, we check that for any variable, adding the IRF of a pure reoptimization shock to the IRF of a shock on G in a timeless perspective (i.e., with no reoptimization) exactly generates the IRF of a shock on G in the time-0 perspective (i.e., with a shock on G plus a reoptimization shock).

[27] An additional benefit of this procedure is that the implied IRFs can be thought as the IRFs of a model with aggregate risk, where we take a first-order approximation of the model for the aggregate risk.

of the SWF and simulate the dynamics of the model. LeGrand and Ragot (2022b) propose a refinement of the truncation method, which we improve in this paper to allow for a refined truncation with an arbitrarily large number of different productivity levels. The formal algebra is detailed in Appendix G.

The truncation method can be summarized as follows. It consists of aggregating the model according to agents' recent idiosyncratic histories, and then expressing the model in terms of these groups of agents rather than individual agents. Indeed, in heterogeneous-agent models, agents differ according to their idiosyncratic histories. An agent i has a period-t history $(y_{i,0}, \ldots, y_{i,t})$. Let $h = (\tilde{y}_{-N+1}, \ldots, \tilde{y}_{-1}, \tilde{y}_0)$ be a given history of length N. In period t, an agent i is said to have *truncated history* h if the history of this agent for the last N periods is equal to h: $(y_{i,t-n+1}, \ldots, y_{i,t}) = (\tilde{y}_{-N+1}, \ldots, \tilde{y}_{-1}, \tilde{y}_0)$. The truncation method then consists of constructing a model based on these truncated histories, which serve as representative agents. The difficulty in the aggregation is that the steady state of the Bewley model features a distribution of agents within each truncated history (according to the history of agents prior to period $t - N$). It can be shown that this within-history heterogeneity can be captured by history-specific parameters (denoted by "ξs"). The truncation method assumes that this within-history heterogeneity is time-invariant and thereby allows the simulation of the dynamics.[28]

The previous truncation method considers truncated histories of equal length. This provides simplicity but at the cost of considering many histories, some of which are very unlikely to be experienced by agents. LeGrand and Ragot (2022b) proposed to consider different truncation lengths for different histories; for clarity, we call this method *refined* truncation and the former one *uniform* truncation. Histories that are more likely to be experienced (i.e., larger histories) can be "refined", i.e., that they can be replaced by a set of histories with a higher truncation length. For instance, the truncated history (y_1, y_1) can be refined into $\{(y, y_1, y_1) : y \in \mathcal{Y}\}$, where the group of agents who have been in productivity y_1 for two consecutive periods is divided into $Card(\mathcal{Y})$ truncated histories, depending on their productivity status 3 periods ago. The construction is recursive because the set $\{(y, y_1, y_1) : y \in \mathcal{Y}\}$ contains the truncated history (y_1, y_1, y_1) that can refined in a similar way. An advantage of this construction is that the number of histories is a *linear* function of the maximum truncation length, instead of an exponential function. A

[28] Considering wealth bins is not possible because the savings function and thus the transitions across wealth bins are endogenous to the planner's policy. This would imply a fixed point that would be very hard to solve. LeGrand and Ragot (2022a) showed that the truncated allocation converges to the true one when the truncation length increases. The question of the truncation length is then quantitative, and LeGrand and Ragot (2023) showed that a tractable truncation length provides accurate results.

difficulty of the construction is that the set of refined histories must form a well-defined partition of the set of idiosyncratic histories in each period. The construction of the refinements is presented in Appendix G.1. One can check the accuracy of the refined truncation, simulating economies where other solution techniques can be used. This is done in Section H, where we use Reiter's (2009) method as a benchmark.

5 Numerical Analysis

5.1 Calibration

Preferences. The period is a quarter. The discount factor (together with the technology parameters) is set to match an annual capital-to-output ratio of 2.7, a standard US estimate. For the log-GHH autility function (32), we set a Frisch elasticity of the labor supply of $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in heterogeneous-agent models. The scaling parameter is set to $\chi = 0.05$ to obtain a steady-state labor supply of roughly 1/3.

Ex-ante types of agents. As explained in Section 4.2, we consider three types of agents that differ ex-ante. Each type of agent is endowed with its own productivity process. The three types will enable the model to replicate at the steady state the actual fiscal policy of the US.

Productivity and idiosyncratic risk. We first use data on US educational attainment to determine the average productivity levels. We set the relative average productivity levels based on the average annual earnings of three groups of workers: those with a high-school degree or less, those with some college education and no college degree or associate degree, and those with at least a bachelor's degree. This leads us to set the relative average productivity levels of the three types to 0.8, 1 and 2, and their corresponding population shares to 1/3 for each type.[29] Type 1 is the type with the lowest average productivity, while Type 3 corresponds to the highest average productivity.

For the labor market process for each type of agents,we follow the strategy of Castaeneda et al. (2003), which is to fit realistic processes based on targeted moments. First, we focus on standard AR(1) processes for each type f: $\log y_t^f = \rho_y^f \log y_{t-1}^f + \varepsilon_t^f$, where

[29]Using the 2022 Current Population Survey, the three groups each represent roughly 1/3 of the US labor force, and have average annual incomes of $24400, $31000 and $71000 respectively.

$\varepsilon_t^f \overset{\text{iid}}{\sim} \mathcal{N}(0, (\sigma_y^f)^2)$. Second, we perform a grid search to minimize the distance between the three processes, while imposing the following constraints: i) consistent with US data (see LeGrand and Ragot, 2018), the agents with the lowest average income face a higher income risk, ii) we target a realistic debt-to-GDP ratio given the chosen fiscal system, and iii) all social weights must be positive. We impose this last constraint to obtain a sensible SWF (in which the planner positively cares for all agents' types). The resulting parameters are gathered in Table 1.

	Type 1	Type 2	Type 3
persistence ρ_y	0.986	0.98	0.98
Variance σ_y	0.16	0.132	0.132
Average productivity	0.8	1.0	2.0

Table 1: Model calibration: targets and model counterparts.

Finally, we discretize each productivity process using the Rouwenhorst (1995) procedure considering five idiosyncratic states for each process. We thus have $5 \times 3 = 15$ productivity levels in the economy.

Technology. The production function is Cobb–Douglas: $F(K, L) = K^\alpha L^{1-\alpha} - \delta K$. The capital share is set to $\alpha = 36\%$ and the depreciation rate to $\delta = 2.5\%$, as in Krueger et al. (2018) among others.

Taxes and government budget constraint. The capital tax is taken from Trabandt and Uhlig (2011), who used the methodology of Mendoza et al. (1994) on public finance data prior to 2008. Their estimation for the US in 2007 (before the financial crisis) yields a capital tax (including both personal and corporate taxes) of $\tau^K = 36\%$. For labor, we consider the HSV functional form from equation (2). The progressivity of the labor tax is taken from Heathcote et al. (2017), who reported an estimate of $\tau = 0.18$. We choose κ to match a public-spending-to-GDP ratio of 17%, as in Heathcote and Tsujiyama (2021).

Table 2 summarizes the model parameters.

5.2 Simulation, Truncation and Estimating SWF Weights

To construct the finite state-space representation, we first use a uniform truncation length of $N = 2$ for each agent, thus generating $15^2 = 225$ histories. Second, we refine the 15

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.992
α	Capital share	0.36
δ	Depreciation rate	2.5%
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.05
φ	Frisch elasticity labor supply	0.5
	Tax system	
τ^K	Capital tax	36%
κ	Scaling of labor tax	0.75
τ	Progressivity of tax	18%
B/Y	Public debt	61%
G/Y	Public consumption	17%

Table 2: Parameter values in the baseline calibration. See the text for descriptions and targets.

most common histories with a truncation length of 10. This means that the total number of histories is 455. This representation provides an accurate simulation of the dynamics, as shown in Appendix H, where we compare the dynamics of the economy simulated with the truncation and the Reiter methods, after a public spending shock for exogenous fiscal rules.

In Appendix G, we provide a detailed account of the computational implementation, which is of independent interest because solving such Ramsey problems is not straightforward. To summarize, the truncation method provides a finite state-space representation, which is used to compute steady-state Lagrange multipliers λ_h and social value of liquidity ψ_h for all histories $h = 1, \ldots, 455$. We then use the method of Section 4.2 to compute the social weights. Because of the two restrictions imposed by the planner's FOCs and the normalization condition, the computation of the SWF weights boils down to the inversion of a 3×3 matrix. The three social weights are found to be: $\omega_1 = 13.1\%$, $\omega_2 = 81.6\%$ and $\omega_3 = 5.3\%$. They are positive and sum to 100% by normalization. We recall that by construction, the chosen fiscal system is optimal for the planner at the steady-state.[30]

[30] We have checked that the social weights move intuitively as a function of the steady-state allocation. For instance, decreasing τ from 0.18 to 0.10 (recall that $\tau = 0$ is a linear fiscal system) and increasing κ to 0.76 to balance the government budget increases the social weight of the most productive agents ($\omega_1 = 5.64\%$, $\omega_2 = 0.70\%$ and $\omega_3 = 0.24\%$). We also checked that increasing the truncation length did not

5.3 Model Dynamics

We now simulate the optimal dynamics of the four fiscal tools $(\tau_t^K, B_t, \kappa_t, \tau_t)_{t\geq 0}$ after a public spending shock occurring in period $t = 0$. After an initial shock denoted ϵ_0 in period 0, public spending reverts back to its equilibrium value at rate ρ_G. The dynamics of public spending are: $G_0 = (1 + \epsilon_0)G_{ss}$ and $G_t = (1 - \rho_G)G_{ss} + \rho_G G_{t-1}$.

Dynamics of the instruments as a function persistence. We simulate the model for two values of the persistence of public spending shocks. The higher value, $\rho_G = 0.99$, corresponds to a very persistent shock. The lower value is $\rho_G = 0.1$ and corresponds to a transitory shock. The initial size of the shock is adjusted so that the NPV of public spending is the same in the two economies. Results are plotted in Figure 1, which reports the public spending shock G, the Lagrange multiplier μ, and optimal public debt B in proportional deviations, and the labor tax level κ, the labor tax progressivity τ, and the capital tax τ^k in level deviations. The high-persistence economy is plotted with blue dashed lines, while the low-persistent one is plotted with black solid lines. The thin red dashed line indicates the zero value (i.e., the steady-state value).

Panel 1 presents the dynamics of public spending, G, which increases by 1% of GDP when $\rho_G = 0.1$ (black solid line) and by 0.02% of GDP when $\rho_G = 0.97$ (blue dashed line). These two different date-0 increases are calculated so that the NPV of public spending is the same in both economies. Panel 6 plots the value of the Lagrange multiplier μ (in proportional deviations), which represents the marginal value of additional public resources. Panels 2–4 report the level of the labor tax, κ, the progressivity of the labor tax, τ, and the capital tax τ^k (in level deviations). Recall that the tax schedule (2) is such that the post-tax wage is $w_t = \kappa_t(\tilde{w}_t)^{1-\tau_t}$. Therefore, an increase in κ (panel 2) corresponds to a decrease in the labor tax (as agents receive more labor income), while an increase in τ (panel 3) implies a more progressive labor tax.

First, after the public spending shock, the capital tax increases (panel 4), and the planner reduces the labor tax (panel 2) and increases its progressivity (panel 3) to reduce income inequality. Note that the change in the capital tax (panel 4) is an order of magnitude larger than the change in the labor tax. Moreover, the higher the persistence, the smaller the change in these variables. However, the variation of taxes at impact as a function of persistence is much lower than the variation in public spending. Indeed, while the tax paths are quite similar for the two persistence levels, the changes in the

significantly change the weights.

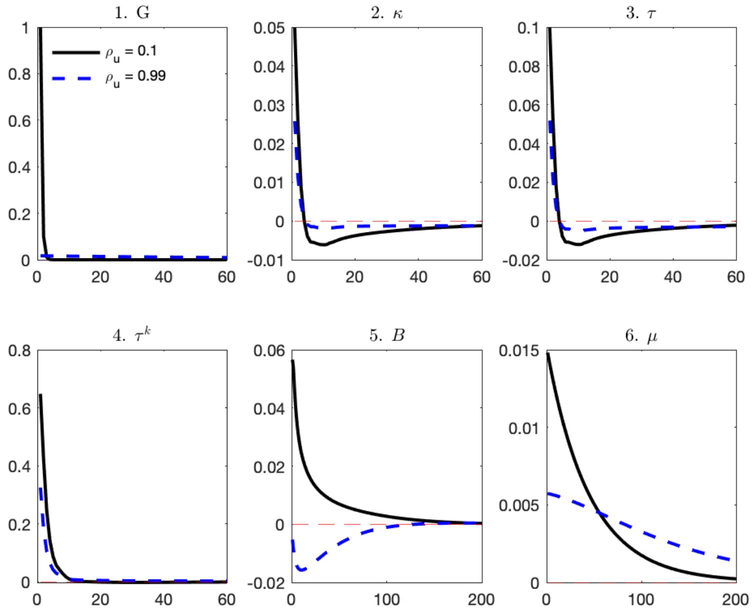

Figure 1: Dynamics of selected variables for two shocks with different persistences and the same NPV. G—public spending; μ—value of public resources; κ—level of labor tax; τ—progressivity of labor tax; τ^k—capital tax; B—public debt. The black solid lines correspond to persistence $\rho_G = 0.1$, and the blue dashed lines correspond to persistence $\rho_G = 0.99$. G is in percent of GDP, B is in proportional deviations, and other variables are in level deviations.

public debt path (panel 5) are quite different. Public debt increases when the persistence is low, making it easier to finance the sharp increase in public spending in the early periods. In contrast, public debt decreases when the persistence is high, as the cost of the additional public spending is front-loaded. These responses of the public debt explain why the variations in the tax responses, functions of the persistence of the public spending shock, are not commensurate with the variations in the date-0 shock.

To summarize, in both cases (high and low persistence), the planner implements a significant increase in capital taxes for a few quarters. Labor taxes move much less, with a small decrease in the overall level and a small increase in progressivity. Public debt shows much more persistent deviations than other variables do. Moreover, it can either fall or rise depending on the persistence of the public spending shock. This confirms the robustness of our theoretical result in Section 3.3, in a quantitatively relevant setting.

Allocation and comparison with the first-best outcome. We now compare the outcomes of the incomplete market model to those of the first-best allocation. The first-best allocation is computed in the complete market economy, in which the planner maximizes aggregate welfare subject only to the resource constraint. The first-best allocation implicitly assumes that the planner has access to productivity-contingent lump-sum taxes, as in the standard real business-cycle model. The weights of the SWF do not affect the dynamics of aggregate quantities in this case, but only the intra-period allocation.

We compare the incomplete market allocation to the first-best one both in terms of aggregate quantities and in terms of per-period welfare expressed in equivalent consumption. The latter is computed as follows. For each type of agent $f = 1, 2, 3$, there are N_{tot} histories indexed by h. We denote by $c^f_{h,t}$ and $l^f_{h,t}$ the consumption and labor supply of agents with history h of type f in period t. Their period welfare, W_t, can be computed as follows:

$$W_t = \sum_{f=1}^{3} \omega^f m^f \sum_{h=1}^{N_{tot}} S^f_h \xi^f_{0,h} u\left(c^f_{h,t} - \frac{1}{\chi} \frac{(l^f_{h,t})^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right),$$

where S^f_h is the share of the population of type f with history h and the parameter $\xi^f_{0,h}$ captures the steady-state heterogeneity within history h of type f. More precisely, $\xi^f_{0,h}$ ensures that the steady-state period utility of each history is equal to the utility derived from steady-state consumption $c^{f,ss}_h$ and labor supply $l^{f,ss}_h$: $\xi_0 u(c^{f,ss}_h \Delta_t - \frac{1}{\chi} \frac{\left(l^{f,ss}_h\right)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}})$ is exactly the steady-state utility of agents with history h of type f in the full model. Using these elements, we can then compute the per-period equivalent consumption, Δ_t, defined as the increase in the steady-state consumption of all agents at time t that makes each agent's period welfare identical to the period welfare W_t. Formally:

$$\sum_{f=1}^{3} \omega^f m^f \sum_{h=1}^{N^{tot}} S^f_h \xi^f_{0,h} u\left(c^{f,ss}_h \Delta_t - \frac{1}{\chi} \frac{\left(l^{f,ss}_h\right)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) = W_t.$$

For the first-best case, the calculation is similar except that there is only one agent (hence one type and one history).

The results are plotted in Figure 2, where we report output Y, capital K, aggregate labor supply L, aggregate consumption C, and welfare (in equivalent consumption as explained above) after a public spending shock. Panel A reports results for the low-persistence case ($\rho_G = 0.1$), and panel B reports results for the high-persistence case ($\rho_G = 0.99$). The public spending shock is shown in panel 1 of Figure 8 for the two persistence values. In both panels of Figure 2, the solid line corresponds to the incomplete market model (IM) and the dashed line to the first-best allocation (FB). The two economies experience the same public spending shock, which differs only in its persistence.

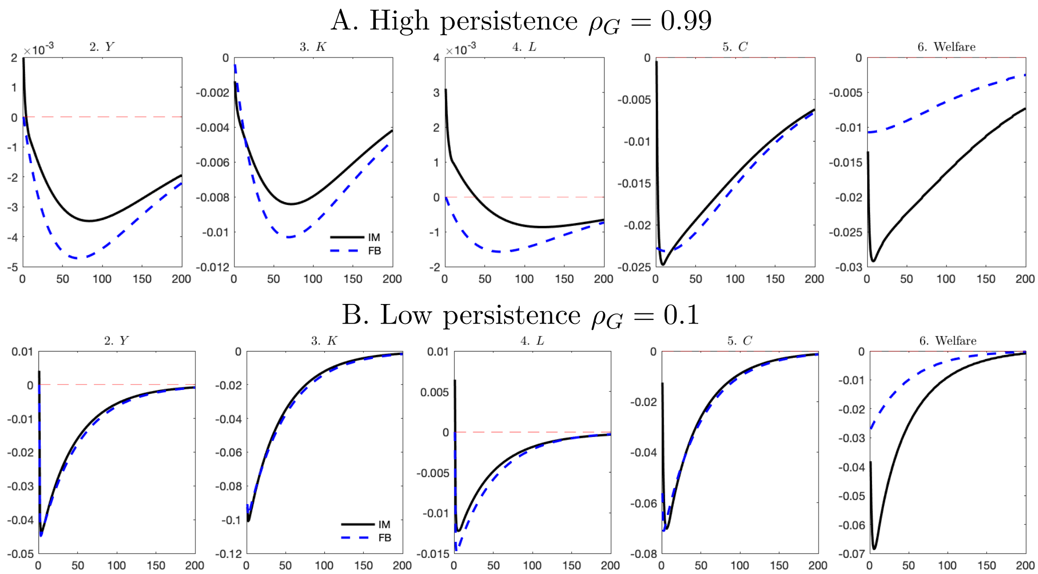

Figure 2: Output Y, capital K, labor L, consumption C, and period aggregate welfare (in equivalent consumption) for low and high persistence values, in proportional deviations. The solid line is the incomplete market model (IM) and the dashed line the first-best allocation (FB). The shock is a public spending shock (panel 1 of Figure 8) and only differs in persistence.

Consumption and the capital stock fall in all cases, but much more so when persistence is low (due to the larger shock at impact). Total labor supply increases at impact (due to a decrease in labor taxes). It can be observed that the volatility and the persistence of the aggregate variables in the incomplete-market economy are higher than in the first-best economy, for both low and high persistence values, although the dynamics of the variables are qualitatively similar.[31] The relative discrepancy between the two economies is greater in the case of high persistence case than in the case of low persistence. Finally, and unsurprisingly, the decline in welfare is significantly more pronounced in the incomplete-market economy than in the first-best economy. Moreover, the welfare gap between the

[31] It is also possible to compute the dynamics of the allocation with complete markets (representative-agent case) but with distorting taxes. It is known (from Chari et al., 1994, Chari and Kehoe, 1999, and Farhi, 2010, among others) that the optimal steady-state outcome features (i) a null capital tax, (ii) a government that holds the whole capital stock (public debt thus being negative), and (iii) a labor tax set to finance the share of public spending that is not financed by interest payment on the capital stock. After a public spending shock, public debt follows the capital stock. Because this outcome is very different from the incomplete-market economy (where steady-state public debt is positive), we do not report the simulation of this economy.

two economies decreases faster in the low persistence case than in the high persistence case.

Optimal path of public debt and persistence of the shock. As we saw in Section 3.3, and as confirmed by the quantitative analysis reported in panel 5 of Figure 1, the response of the public debt differs markedly with the persistence of the shock. We explore this aspect further here by reporting the optimal debt dynamics for four different levels of persistence of the public spending shock. This is done in Figure 3, where in each case the initial shock G_0 is normalized to produce the same NPV of public spending. The paths of other instruments or other aggregate quantities are similar to those presented in Figure 1 and are therefore not reported here.

We observe that the response of the public debt at impact decreases with persistence. When the persistence is small ($\rho_G = 0.1$), public debt on impact first increases and then decreases monotonically. The response at impact decreases monotonically with persistence. The shape of the response also changes. For higher persistence ($\rho_G = 0.8$), the path of public debt has an inverted U shape that then becomes J-shaped at higher persistence ($\rho_G = 0.95$).

The takeaway from this graph is that the persistence of the public spending shock is a key driver of the optimal financing structure of the shock. The higher the persistence, the more the financing should rely on taxes and the less on public debt.

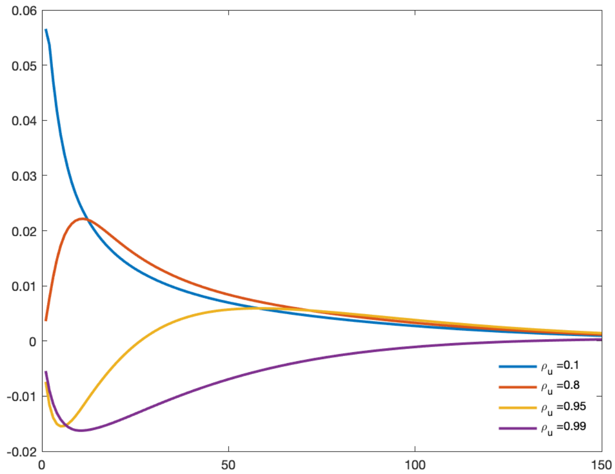

Figure 3: Comparison of optimal public debt dynamics for different persistence values of the public shock (same NPV of public spending), in proportional deviation from steady-state value of public debt.

5.4 Robustness in Other Environments and Other Shocks

We check the robustness of the result concerning the optimal response of public debt in two other environments.

5.4.1 Alternative Fiscal System

Our benchmark tax system features a non-linear HSV labor tax. However, this is not the only way in the literature to reproduce the progressivity of the US fiscal system. Another possibility is to consider an affine tax system, in which the linear labor tax is complemented by a lump-sum transfer T_t. This is the case, for instance in Dyrda and Pedroni (2022). We present the model specification and the solution of the Ramsey program in Appendix I. We verify that our main result remains robust to this new fiscal system. We find that public debt increases when persistence is low but decreases when persistence is high. In addition, similarly to the quantitative results in Figure 1, both the tax progressivity and the capital tax increase at impact. Overall, the optimal response of the fiscal system remains robust to the specification of the tax system.

5.4.2 Alternative SWF

The benchmark SWF assigns social weights that depend on the ex-ante type of agents. The ex-ante types were fixed once and for all, and implied different productivity processes among agents. We then used these weights to implement an inverse optimal taxation approach: the weights were calibrated for the actual US tax system to be optimal at the steady state. Here we consider an alternative SWF, where the weights are productivity dependent. We assume that agents can draw their current level of productivity within a given finite set and the planner then assigns a social welfare weight, $\omega(y)$, to each level of productivity, y. The instantaneous utility of agents with productivity level y in the current period is weighted by $\omega(y)$. Even if the weights are set once and for all, a given agent may experience different weights depending on their current productivity level. Formally, the planner's SWF is:

$$\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_{i,t})(u(c_{i,t}) - v(l_{i,t}))\ell(di), \tag{65}$$

where we remove ex-ante heterogeneity. In fact, this formulation of social welfare weights does not require ex-ante heterogeneity, since it only involves within-period heterogeneity. The advantage of the representation (65), is that the planner may have a preference for within-period redistribution, since the weights depend on productivity, which may be an attractive feature. This SWF is used in LeGrand et al. (2022), Dávila and Schaab (2022) and McKay and Wolf (2023). This representation also allows considering a separable utility function, which is CRRA for consumption. However, these weights are not strictly speaking social weights, since they weight instantaneous utility but not intertemporal welfare.

The model specification and results are provided in Appendix J. Again, we check that the results are qualitatively similar to those of the baseline model.

5.4.3 Other Shocks

The dynamics of the model with other shocks can be simulated using the same approach as in Section 5.2. In fact, once the Ramsey steady state has been characterized, the method for simulating the dynamics of the model is very versatile and can be easily adapted to different shocks. We illustrate this here by considering a TFP shock and a discount factor shock that complements the aforementioned public spending shock.

Regarding the TFP shock, we assume that the production function is $Y_t = F(K_{t-1}, L_t) = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1}$, where Z_t is the TFP, equal to 1 in the steady-state. Public spending is constant. After an initial (small) shock ϵ_0, TFP returns to its equilibrium value at a rate $\rho_Z \in [0, 1)$. The dynamics of TFP is therefore: $Z_0 = 1 - \epsilon_0$ and $Z_t = 1 - \rho_Z + \rho_Z Z_{t-1}$, for $t \geq 1$. We then simulate the dynamics of the model for different values of the persistence ρ_Z. As in the case of the public spending shock, we adjust the size of the initial shock ϵ_0, so that the cumulative fall in TFP is constant over the different values of ρ_Z. For the sake of brevity, the optimal dynamics of public debt for the different values of ρ_Z can be found in Appendix K. The results are very similar to those in Figure 3. When the TFP shock is not persistent, the public debt increases on impact, while it decreases when it is very persistent. The intuition behind this result is as follows. An increase in public spending is a reduction in the resources available for consumption (even if it increases the level of welfare), which is known to be very similar to a fall in TFP. In both cases, the planner must raise additional resources, either because spending increases or because the tax bases shrink.

Regarding the discount factor shock, the discount factor shared by agents and the planner is time-varying, while public spending and TFP remain constant. Formally, the path of the discount factor, denoted $(\beta_t)_{t\geq 0}$, is defined as $\beta_0 = \beta + \epsilon_0$ and $\beta_t = (1 - \rho_\beta)\beta + \rho_\beta \beta_{t-1}$, for $t \geq 1$. The initial shock is ϵ_0 and the persistence $\rho_\beta \in [0, 1)$, while β is the steady-state value of the discount factor set using the baseline calibration. Again, we consider the public debt response for different values of the persistence ρ_β and the initial shock is normalized so that the average variation in the discount factor is constant across the different persistence values. The results can be found in Appendix K. We find that the dynamics of the public debt is different compared to the cases of the TFP and public spending shocks. Since agents are temporarily more patient, the capital stock increases and this increase is more persistent than the discount factor shock. This allows the planner to reduce the public debt. When the persistence is very high, public debt may even increase slightly on impact, since the high persistence implies a large and sustained increase in savings.

6 Conclusion

We have studied the optimal fiscal policy after a public spending shock in a heterogeneous-agent model. Our first contribution is to clarify, in a simple environment, the conditions for the existence of steady-state equilibria that feature positive optimal capital taxation and public debt. The key friction for the existence of an equilibrium is an occasionally binding credit constraint, which provides a rationale for maintaining both a positive capital tax and positive public debt. This friction is necessary but not sufficient for the existence of the equilibrium. Obtaining a positive optimal capital tax depends on the shape of the utility function; it occurs generally for DRRA or GHH utility functions, for instance. A second result is to show that the optimal dynamics of public debt and taxes depend crucially on the persistence of the public spending shock: public debt is procyclical for low persistence but countercyclical for high persistence. We show that these results still hold in a general model where we solve for optimal fiscal policy after an MIT shock. In this model, the actual US tax system is optimally implemented at the steady state, thanks to an inverse optimal taxation approach. We find that public debt can either increase or decrease on impact depending of the persistence of public spending or TFP shocks.

References

Açikgöz, O., M. Hagedorn, H. Holter, and Y. Wang (2022): "The Optimum Quantity of Capital and Debt," Working Paper, University of Oslo.

Acharya, S., E. Challe, and K. Dogra (2023): "Optimal Monetary Policy According to HANK," *American Economic Review*, 113, 1741–1782.

Acharya, S. and K. Dogra (2021): "Understanding HANK: Insights from a PRANK," *Econometrica*, 88, 1113–1158.

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

——— (1995): "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 103, 1158–1175.

Aiyagari, S. R. and E. R. McGrattan (1998): "The Optimum Quantity of Debt," *Journal of Monetary Economics*, 42, 447–469.

Albanesi, S. and R. Armenter (2012): "Intertemporal Distortions in the Second Best," *The Review of Economic Studies*, 79, 1271–1307.

Auclert, A., M. Cai, M. Rognlie, and L. Straub (2022): "Optimal Long-Run Fiscal Policy with Heterogeneous Agents," SED Presentation.

Barro, R. J. (1979): "On the Determination of the Public Debt," *Journal of Political Economy*, 87, 940–971.

Bassetto, M. (2014): "Optimal Fiscal Policy with Heterogeneous Agents," *Quantitative Economics*, 5, 675–704.

Bassetto, M. and W. Cui (2021): "A Ramsey Theory of Financial Distortions," IFS Working Paper WP202105, Institute for Fiscal Studies.

Bayer, C., R. Luetticke, L. Pham-Dao, and V. Tjaden (2019): "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," *Econometrica*, 87, 255–290.

Bewley, T. F. (1983): "A Difficulty with the Optimum Quantity of Money," *Econometrica*, 51, 1485–1504.

Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2017): "Public Debt in Economies with Heterogeneous Agents," *Journal of Monetary Economics*, 91, 39–51.

——— (2021): "Inequality, Business Cycles, and Monetary-Fiscal Policy," *Econometrica*, 89, 2559–2599.

Bohn, H. (1998): "The Behavior of US Public Debt and Deficits," *Quarterly Journal of Economics*, 4, 1329–1368.

Boppart, T., P. Krusell, and K. Mitman (2018): "Exploiting MIT Shocks in Heterogeneous-Agent Economies: The Impulse Response as a Numerical Derivative," *Journal of Economic Dynamics and Control*, 89, 68–92.

Bourguignon, F. and S. Amadeo (2015): "Tax-Benefit Revealed Social Preferences," *Journal of Economic Inequality*, 1, 75–108.

Castaeneda, A., J. Diaz-Gimenez, and J.-V. Rios-Rull (2003): "Accounting for Earnings and Wealth Inequality," *Journal of Political Economy*, 111, 818–857.

Challe, E. and X. Ragot (2016): "Precautionary Saving over the Business Cycle," *Economic Journal*, 126, 135–164.

Chamberlain, G. and C. Wilson (2000): "Optimal Intertemporal Consumption under Uncertainty," *Review of Economic Dynamics*, 3, 365–395.

Chamley, C. (1986): "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, 607–622.

Chang, B.-H., Y. Chang, and S.-B. Kim (2018): "Pareto Weights in Practice: A Quantitative Analysis Across 32 OECD Countries," *Review of Economic Dynamics*, 28, 181–204.

Chari, V., L. Christiano, and P. Kehoe (1994): "Optimal Fiscal Policy in a Business Cycle Model," *Journal of Political Economy*, 102, 617–652.

Chari, V. V. and P. J. Kehoe (1999): "Optimal Fiscal and Monetary Policy," in

Handbook of Macroeconomics, ed. by J. B. Taylor and M. Woodford, Elsevier, vol. 1, chap. 21, 1671–1745.
Chari, V. V., J.-P. Nicolini, and P. Teles (2020): "Optimal Capital Taxation Revisited," *Journal of Monetary Economics*, 116, 147–165.
Chen, Y., Y. Chien, and C. Yang (2020): "Implementing the Modified Golden Rule? Optimal Ramsey Capital Taxation with Incomplete Markets Revisited," St Louis Fed Working Paper 2017-003I, Federal Reserve Bank of St Louis.
Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence On the Intensive and Extensive Margins," *American Economic Review*, 101, 471–475.
Chien, Y. and Y. Wen (2023): "The Ramsey Steady-State Conundrum in Heterogeneous-Agent Economies," St Louis Fed Working Paper 2022-009C, Federal Reserve Bank of St Louis.
Collard, F., M. Angeletos, and H. Dellas (2023): "Public Debt As Private Liquidity: Optimal Policy," *Journal of Political Economy*, 131, 3233–3264.
Conesa, J. C., S. Kitao, and D. Krueger (2009): "Taxing Capital? Not a Bad Idea after All!" *American Economic Review*, 99, 25–48.
Dávila, E. and A. Schaab (2022): "Welfare Assessments with Heterogeneous Individuals," Working Paper 30571, National Bureau of Economic Research.
Dávila, J., J. H. Hong, P. Krusell, and J.-V. Ríos-Rull (2012): "Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks," *Econometrica*, 80, 2431–2467.
Diamond, P. (1998): "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates," *American Economic Review*, 88, 83–95.
Dyrda, S. and M. Pedroni (2022): "Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks," *Review of Economic Studies*, 90, 744–780.
Farhi, E. (2010): "Capital Taxation and Ownership When Markets Are Incomplete," *Journal of Political Economy*, 118, 908–948.
Feldstein, M. (1985): "Debt and Taxes in the Theory of Public Finance," *Journal of Public Economics*, 28, 233–245.
Ferriere, A. and G. Navarro (2023): "The Heterogeneous Effects of Government Spending: It's All About Taxes," *Review of Economic Studies*, forthcoming.
Fishburn, P. (1977): "Mean-Risk Analysis with Risk Associated with Below-Target Returns," *American Economic Review*, 67, 116–126.
Floden, M. (2001): "The Effectiveness of Government Debt and Transfers as Insurance," *Journal of Monetary Economics*, 48, 81–108.
Green, E. (1994): "Individual-Level Randomness in a Nonatomic Population," Working Paper, University of Minnesota.
Heathcote, J. (2005): "Fiscal Policy with Heterogeneous Agents and Incomplete Markets," *Review of Economic Studies*, 72, 161–188.
Heathcote, J., K. Storesletten, and G. L. Violante (2017): "Optimal Tax Progressivity: An Analytical Framework," *Quarterly Journal of Economics*, 132, 1693–1754.
Heathcote, J. and H. Tsujiyama (2021): "Optimal Income Taxation: Mirrlees Meets Ramsey," *Journal of Political Economy*, 129, 3141–3184.
Huggett, M. (1993): "The Risk Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17, 953–969.
İmrohoroğlu, A. (1989): "Cost of Business Cycles with Indivisibilities and Liquidity

Constraints," *Journal of Political Economy*, 97, 1364–1383.
Judd, K. L. (1985): "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 28, 59–83.
King, R., C. Plosser, and S. Rebelo (1988): "Production, Growth and Business Cycles I. The Basic Neoclassical Model," *Journal of Monetary Economics*, 21, 195–232.
Krueger, D., K. Mitman, and F. Perri (2018): "On the Distribution of the Welfare Losses of Large Recessions," in *Advances in Economics and Econometrics: Volume 2, Eleventh World Congress of the Econometric Society*, ed. by B. Honoré, A. Pakes, M. Piazzesi, and L. Samuleson, Cambridge University Press, 143–184.
Krusell, P. and A. A. J. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896.
Lansing, K. J. (1999): "Optimal Redistributive Capital Taxation in a Neoclassical Growth Model," *Journal of Public Economics*, 73, 423–453.
LeGrand, F., A. Martin-Baillon, and X. Ragot (2022): "Should Monetary Policy Care About Redistribution? Optimal Fiscal and Monetary Policy with Heterogeneous Agents," Working paper, SciencesPo.
LeGrand, F. and X. Ragot (2018): "A Class of Tractable Incomplete-Market Models for Studying Asset Returns and Risk Exposure," *European Economic Review*, 103, 39–59.
——— (2022a): "Managing Inequality Over the Business Cycle: Optimal Policies with Heterogeneous Agents and Aggregate Shocks," *International Economic Review*, 63, 511–540.
——— (2022b): "Refining the Truncation Method to Solve Heterogeneous-Agent Models," *Annals of Economics and Statistics*, 146, 65–91.
——— (2023): "Optimal Policies with Heterogeneous Agents: Truncation and Transitions," *Journal of Economic Dynamics and Control*, 156, 104737.
Ljungqvist, L. and T. J. Sargent (2018): *Recursive Macroeconomic Theory*, MIT Press, fourth ed.
Marcet, A. and R. Marimon (2019): "Recursive Contracts," *Econometrica*, 87, 1589–1631.
McKay, A. and C. Wolf (2023): "Optimal Policy Rules in HANK," Working Paper, FRB Minneapolis.
Mendoza, E., A. Razin, and L. Tesar (1994): "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption," *Journal of Monetary Economics*, 34, 297–323.
Nuño, G. and C. Thomas (2022): "Optimal Redistributive Inflation," *Annals of Economics and Statistics*, 146, 3–64.
Reinhorn, L. (2019): "On optimal redistributive capital taxation," *Journal of Public Economic*, 21, 460–487.
Reiter, M. (2009): "Solving Heterogeneous-Agent Models by Projection and Perturbation," *Journal of Economic Dynamics and Control*, 33, 649–665.
Rohrs, S. and C. Winter (2017): "Reducing Government Debt in the Presence of Inequality," *Journal of Economic Dynamics and Control*, 82, 1–20.
Rouwenhorst, G. K. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *Structural Models of Wage and Employment Dynamics*, ed. by T. Cooley, Princeton: Princeton University Press, 201–213.
Slater, M. (1950): "Lagrange Multipliers Revisited," Working Paper 80, Cowles Foundation, reprinted in Giorgi G., Kjeldsen, T., eds. (2014). Traces and Emergence of Nonlinear Programming. Basel.

Straub, L. and I. Werning (2020): "Positive Long Run Capital Taxation : Chamley-Judd Revisited," *American Economic Review*, 110, 86–119.

Trabandt, M. and H. Uhlig (2011): "The Laffer Curve Revisited," *Journal of Monetary Economics*, 58, 305–327.

Woodford, M. (1990): "Public Debt as Private Liquidity," *American Economic Review*, 80, 382–388.

Appendix

A Proof of Proposition 1

When credit constraints are binding, the simplest solution to compute the planner's FOCs is to use the primal approach. These FOCs can also be derived using either the factorization approach or the Lagrangian approach, as explained in Section 4.1. See Appendix F.2.1 below for the derivation. The Lagrangian associated to the program (17)–(18) can be written as:

$$\max_{(c_{e,t},l_{e,t},c_{u,t},a_{e,t},B_t)_t} \sum_{t=0}^{\infty}\beta^t\Big(U(c_{e,t},l_{e,t})+U(c_{u,t},0)-\lambda_{e,t}(w_t l_{e,t}-(a_{e,t}+c_{e,t}))-\lambda_{u,t}(R_t a_{e,t-1}-c_{u,t})$$

$$-\mu_t(c_{e,t}+c_{u,t}+G_t+a_{e,t}-B_t-a_{e,t-1}+B_{t-1}-F(a_{e,t-1}-B_{t-1},l_{e,t}))\Big), \quad (66)$$

$$\text{s.t. } w_t = \frac{-U_l(c_{e,t},l_{e,t})}{U_c(c_{e,t},l_{e,t})}, \quad (67)$$

$$R_{t+1} = \frac{U_c(c_{e,t},l_{e,t})}{\beta U_c(c_{u,t+1},0)}, \quad (68)$$

$$c_{e,t}, c_{u,t} > 0,\ a_{e,t}, l_{e,t} \geq 0, \quad (69)$$

where, to simplify expressions, we consider the prices as functions of allocations: $w_t := w_t(l_{e,t}, c_{e,t})$ and $R_{t+1} := R_{t+1}(l_{e,t}, c_{e,t}, c_{u,t+1})$, and we will thus write partial derivatives, such as $\frac{\partial \log w_t}{\partial c_{e,t}}$. These derivatives reflect the externalities of allocation choices on prices that are internalized by the planner. The quantities $\lambda_{e,t}$ and $\lambda_{u,t}$ are the Lagrange multiplier associated to the resource constraints of employed and unemployed, respectively. We follow Ljungqvist and Sargent (2018) in this formulation of the primal approach (see below for a lengthier discussion).[32]

The FOCs of the planner with respect to B_t and $a_{e,t}$ imply:

$$\mu_t = \beta(1+F_{K,t+1})\mu_{t+1}, \quad (70)$$

$$\lambda_{e,t} = \beta R_{t+1}\lambda_{u,t+1}. \quad (71)$$

The FOC (70) means that the planner smooths out the shadow cost of its budget constraint. Relaxing it today by one unit implies constraining it tomorrow by $1 + F_{K,t+1}$, which is the

[32] In particular, a positive $\lambda_{e,t}$ or $\lambda_{u,t}$ does not imply that relaxing the agent's budget constraint negatively affects welfare, as the actual Lagrange multipliers on agents' budget constraints are $\mu_t - \lambda_{e,t}$ and $\mu_t - \lambda_{u,t}$, which are always positive.

cost of public debt. This equation at the steady state implies the modified golden rule stating that the planner chooses the long-run level of public debt so as to equalize the marginal productivity of capital and discount rate. The FOC (71) states that relaxing the employed budget constraint in the current period implies constraining the unemployed budget constraint in the next period. The shadow costs of budget constraints are discounted as the Euler equation, with the discount factor βR_{t+1}.

The FOCs with respect to $c_{e,t}$, $c_{u,t}$, and $l_{e,t}$ are respectively:

$$0 = U_{c,e,t} - \mu_t - \lambda_{e,t}\left(\frac{\partial \log w_t}{\partial c_{e,t}} w_t l_{e,t} - 1\right) - \beta\lambda_{u,t+1}\frac{\partial \log R_{t+1}}{\partial c_{e,t}} R_{t+1} a_{e,t}, \tag{72}$$

$$0 = U_{c,u,t} - \mu_t - \lambda_{u,t}\left(\frac{\partial \log R_t}{\partial c_{u,t}} R_t a_{e,t-1} - 1\right), \tag{73}$$

$$0 = U_{l,e,t} + \mu_t F_{L,t} - \lambda_{e,t}\left(w_t + \frac{\partial \log w_t}{\partial l_{e,t}} w_t l_{e,t}\right) - \beta\lambda_{u,t+1}\frac{\partial \log R_{t+1}}{\partial l_{e,t}} R_{t+1} a_{e,t}, \tag{74}$$

where $U_{c,e,t} := \frac{\partial U(c_{e,t}, l_{e,t})}{\partial c_{e,t}}$ and similarly for other notation. Multiplying (73) at date $t+1$ by βR_{t+1} and subtracting (72) yields using (68), (70) and (71):

$$\begin{aligned} &\mu_t - \beta R_{t+1}\mu_{t+1} = \beta(1 + F_{K,t+1} - R_{t+1})\mu_{t+1} = \beta\tau^K_{t+1}\tilde{r}^K_{t+1}\mu_{t+1} = \\ &\lambda_{e,t}\left(c_{u,t+1}\frac{\partial \log R_{t+1}}{\partial c_{u,t+1}} - \frac{c_{u,t+1}}{R_{t+1}}\frac{\partial \log R_{t+1}}{\partial c_{e,t}} - w_t l_{e,t}\frac{\partial \log w_t}{\partial c_{e,t}}\right), \end{aligned} \tag{75}$$

which can be interpreted as the planner setting jointly consumption levels of employed and unemployed in a way that is similar to an increase in the savings of employed agents at date t. The consumption of employed agents at t decreases by one unit, while the consumption of unemployed at $t+1$ increases by R_{t+1} units. Equation (75) sets equal the benefit in terms of resources (proportional to the Lagrange multiplier μ) of the to its social cost (proportional to the shadow cost of individual budget constraint λ_e). Decreasing the consumption of employed agents at t by one unit has a benefit μ_t (or $\beta(1 + F_{K,t+1})\mu_{t+1}$ from (70)) for resources, while increasing the consumption of unemployed at $t+1$ by R_{t+1} units has a today's cost of $\beta\mu_{t+1}$ per unit. The net benefit at the left hand side of (75) is thus proportional to $1 + F_{K,t+1} - R_{t+1}$, which is itself proportional to the capital tax. This comes at no surprise: increasing savings raises the capital tax base, which makes the resource benefit proportional to the capital tax. The latter reflects the *smoothing wedge* due to the difference in discount rate between the planner and agents. Ideally, the planner would like to raise savings up to the point where the marginal benefit is null and the benefit maximal. It does not do so because raising savings also involves a cost.

The cost, at the right hand side of (75), is due to the (price) externalities of savings on wages and interest rates. Indeed, there is no direct effect, because the planner chooses a savings level consistent with the Euler equation (68) and the envelope theorem applies. The magnitude of the price externality is controlled by the Lagrange multiplier $\lambda_{e,t}$, while the externality itself is composed of the three terms between brackets in (75). The first two terms are *savings* channels and reflect how the change in consumption affect interest rate. Each effect is proportional to the "effect base" (i.e., the consumption level) and to the semielasticity of interest rate to the relevant consumption. The last term is the *labor* channel and reflects how the change in savings affect the wage. This term is proportional to the total wage, $w_t l_{e,t}$, and to the semielasticity of the wage to the employed consumption.

Similarly to (75), multiplying (72) by w_t and adding (74) yields using (67) and (71):

$$(F_{L,t}-w_t)\mu_t = \lambda_{e,t}\left(w_t l_{e,t}\Big(w_t\frac{\partial \log w_t}{\partial c_{e,t}}+\frac{\partial \log w_t}{\partial l_{e,t}}\Big)+\frac{c_{u,t+1}}{R_{t+1}}\Big(w_t\frac{\partial \log R_{t+1}}{\partial c_{e,t}}+\frac{\partial \log R_{t+1}}{\partial l_{e,t}}\Big)\right) \tag{76}$$

which equalizes the net benefit of raising the labor supply by one unit (and hence the consumption of employed agents by w_t units) in terms of resources to its social cost. The net benefit for resources (proportional to μ_t) is equal to the difference between the marginal increase in GDP due to the higher labor supply and the marginal increase in employed consumption (by w_t units). This benefit is also proportional to the labor tax. Ideally, absent of any other cost, the planner would raise labor supply up to the point where the wage is equal to the marginal productivity of labor, which would maximize planner's resources. The planner does not do so because of the cost related to setting the labor supply. As for savings, there is no direct cost because of the FOC on labor supply. The cost channels through the externality of labor supply on prices. The magnitude of the externality is driven by the Lagrange multiplier $\lambda_{e,t}$ and the externality itself is composed of the sum of two main terms. The first set of terms is proportional to the total wage, $w_t l_{e,t}$, and involves the semielasticities of the wage with respect to labor supply and consumption. The second set of term is proportional to the unemployed consumption and involves the semielasticities of the interest rate with respect to labor and consumption.

The semielasticities are at the core of the relationships (75) and (76). The Euler equation (68) and the labor FOC (67) imply:

$$\frac{\partial \log R_{t+1}}{\partial c_{e,t}} = \frac{U_{cc,e,t}}{U_{c,e,t}}, \; \frac{\partial \log R_{t+1}}{\partial c_{u,t+1}} = -\frac{U_{cc,u,t+1}}{U_{c,u,t+1}}, \; \frac{\partial \log R_{t+1}}{\partial l_{e,t}} = \frac{U_{cl,e,t}}{U_{c,e,t}},$$
$$\frac{\partial \log w_t}{\partial c_{e,t}} = \frac{U_{cl,e,t}}{U_{l,e,t}} - \frac{U_{cc,e,t}}{U_{c,e,t}}, \; \frac{\partial \log w_t}{\partial l_{e,t}} = \frac{U_{ll,e,t}}{U_{l,e,t}} - \frac{U_{cl,e,t}}{U_{c,e,t}}.$$

Substituting these relationships into (75) and (76) yields:

$$\beta \tau^K_{t+1} \tilde{r}^K_{t+1} \mu_{t+1} = \lambda_{e,t} \left(c_{e,t} \frac{U_{cc,e,t}}{U_{c,e,t}} - c_{u,t+1} \frac{U_{cc,u,t+1}}{U_{c,u,t+1}} + l_{e,t} \frac{U_{cl,e,t}}{U_{c,e,t}} \right), \tag{77}$$
$$\mu_t \tau^L_t \frac{\tilde{w}_t}{w_t} = \lambda_{e,t} \left(l_{e,t} \frac{U_{ll,e,t}}{U_{l,e,t}} - l_{e,t} \frac{U_{cl,e,t}}{U_{c,e,t}} + c_{e,t} \frac{U_{cl,e,t}}{U_{l,e,t}} - c_{e,t} \frac{U_{cc,e,t}}{U_{c,e,t}} \right). \tag{78}$$

At the steady state, and assuming that the Lagrange multipliers μ and λ_e are finite (in our standard SRE), we find equation (29) and the subsequent equations of Section 3.1 with $\Xi = \frac{\lambda_e}{\beta\mu}$.

B Existence Results for Separable Utility Functions

We consider utility functions of the form $U(c,l) = u(c) - v(l)$.

B.1 DRRA Utility Function

B.1.1 The Stone-Geary Utility Function

The Stone-Geary utility function is DRRA and corresponds to $u(c) = \frac{(c-\underline{c})^{1-\sigma}-1}{1-\sigma}$ if $\sigma \neq 1$ or $\log(c-\underline{c})$ otherwise. The term $\underline{c}$ is a minimum consumption level, and $\sigma > 0$ is the inverse of the elasticity of substitution. For the sake of convenience, we assume $v(l) = \chi^{-1}\frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$, where $\varphi > 0$ is the Frisch elasticity of labor supply, and $\chi > 0$ scales labor disutility. In this case, we have: $\sigma_e = \frac{\sigma c_e}{c_e - \underline{c}}$, $\sigma_u = \frac{\sigma c_u}{c_u - \underline{c}}$, $\varphi_e = \varphi$, $\varsigma_e = 0$, whose expressions can be plugged into (29) to obtain the wedge relationship.

Rather than further algebra derivation, we provide a numerical example of a Ramsey equilibrium with positive taxes and binding credit constraints. We consider the calibration of Table 3. The parameters (σ, β, α, and δ) are set to standard values. The preference parameter $\underline{c}$ is set to 1.

In the equilibrium with binding credit constraints for unemployed agents, this calibration generates the allocation and prices described in Table 4 – where we do not repeat that for

Parameters	Value
discount factor β	0.96
capital share α	0.36
capital depreciation rate δ	0.025
inverse of IES σ	1.0
labor scaling factor χ	1.0
Frish elasticity φ	0.5
utility consumption threshold $\underline{c}$	1.0
steady-state public spending G	4.3278

Table 3: Calibration of an economy with a Stone-Geary utility function.

unemployed, labor supply and asset holdings are null. With a DRRA utility function, all taxes and the NDG are positive. The equilibrium with binding credit constraint exists.[33]

Allocation and prices		
employed agents	consumption c_e	1.087
	labor supply l_e	2.910
unemployed agents	consumption c_u	1.085
taxes	capital tax τ^K	0.626
	labor tax τ^L	0.552

Table 4: Allocation of the economy with the calibration of Table 3.

B.1.2 The Fishburn Utility Function

The utility function proposed in Fishburn (1977), is isoelastic below a threshold and linear after it. More formally:

$$u(c) = \begin{cases} \underline{c}^{\sigma} \frac{c^{1-\sigma} - \underline{c}^{1-\sigma}}{1-\sigma} \text{ if } \sigma \neq 1 \text{ or } \underline{c} \log\left(\frac{c}{\underline{c}}\right) \text{ otherwise} & \text{if } 0 < c \leq \underline{c}, \\ c - \underline{c} & \text{if } \underline{c} \leq c, \end{cases}$$

where $\underline{c} > 0$ is a threshold. The function u is continuously differentiable on $\mathbb{R}_+^*$. This utility function was used by Challe and Ragot (2016) and LeGrand and Ragot (2018) because it generates tractable models. We again assume $v(l) = \chi^{-1} \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$. Assuming $c_e > \underline{c} > c_u$,

[33] We have also checked that: (i) the public spending is too high for the first-best equilibrium to exist, (ii) the calibration fulfills the Blanchard-Kahn conditions, and (iii) Proposition 5 still holds. We also did so for other specifications (Fishburn, CARA, and KPR), even though we do not mention it below.

which must be checked in equilibrium, we have: $\sigma_e = \sigma$, $\sigma_u = 0$, $\varphi_e = \varphi$, $\varsigma_e = 0$, that can be plugged into (29) to obtain a relationship between capital and labor taxes.

As in the Stone-Geary case, we provide a numerical example rather than algebra derivation. We consider the calibration of Table 5. The other parameters (β, α, δ, σ, χ, φ) are identical to those of Table 3.

Parameters	utility consumption threshold $\underline{c}$	public spending G
Value	1.0	0.5

Table 5: Calibration with a Fishburn utility function. Other parameters as in Table 3.

This calibration generates the allocation and prices described in Table 6. Consumption levels are consistent with the threshold $\underline{c}$, since $c_u < \underline{c} < c_e$. Tax rates are positive, and the post-tax gross rate R verifies $0 < \beta R < 1$. Since the function is DRRA, the equilibrium features positive taxes and positive NDG, as in the Stone-Geary case. The equilibrium with binding credit constraint exists.

Allocation and taxes		
employed agents	consumption c_e	0.934
	labor supply l_e	1.171
unemployed agents	consumption c_u	0.697
taxes	capital tax τ^K	9.160%
	labor tax τ^L	0.007%

Table 6: Allocation in the economy with the calibration of Table 5.

B.2 An Example of IRRA Utility Function: The CARA Case

A standard example of IRRA utility function is the CARA case, which corresponds to the utility function $u(c) = -\frac{1}{\gamma}e^{-\gamma c}$. We also assume $v(l) = \frac{1}{\chi\varphi}e^{\varphi l}$, where $\gamma, \varphi > 0$. We then have: $\sigma_e = \gamma c_e$, $\sigma_u = \gamma c_u$, $\varphi_e = (\varphi l)^{-1}$, $\varsigma_e = 0$, where $\sigma_e > \sigma_u$ if $c_e > c_u$.

We provide a numerical example of an existing equilibrium with binding credit constraint. We use the calibration of Table 7. The other parameters (β, α, δ, and φ) are identical to those of Table 5. Note that $\underline{c}$ does not play any role in this case.

The allocation featuring positive capital taxes is summarized in Table 8. The equilibrium with binding credit constraint for unemployed agents thus exists with CARA utility function.

Parameters	absolute risk aversion γ	labor scaling factor χ	public spending G
Value	1.0	0.58811	0.01

Table 7: Calibration with a CARA utility function. Other parameters as in Table 5.

A particularity of this equilibrium comes from the IRRA property of CARA utilities, which implies a negative NDG and hence a negative labor tax.

Allocation and prices		
employed agents	consumption c_e	0.160
	labor supply l_e	0.139
unemployed agents	consumption c_u	0.141
taxes	capital tax τ^K	0.473
	labor tax τ^L	-0.295

Table 8: Allocation of the economy with the calibration of Table 7.

B.3 The KPR Utility Function

For the sake of completeness, we apply the same analysis to another standard non-separable utility function, which is the KPR utility function (King et al., 1988). This utility function is for instance used by Dyrda and Pedroni (2022) to compute optimal tax rates. We use the following standard functional form: $U(c,l) = \frac{1}{1-\sigma} c^{\gamma(1-\sigma)}(1-l)^{(1-\gamma)(1-\sigma)}$ and $U(c,l) = \gamma \log(c) + (1-\gamma)\log(1-l)$ if $\sigma = 1$. In this case, the IES is $\frac{1}{1-\gamma+\gamma\sigma}$.

The labor FOC can also be written as: $c_{e,t} = \frac{\gamma}{1-\gamma} w_t (1 - l_{e,t})$. Combined with the budget constraint of employed agents, we obtain: $\frac{1-l_{e,t}}{1-\gamma} = 1 - \frac{a_{e,t}}{w_t}$. The constraints $c_{e,t} \geq 0$ and $a_{e,t} \geq 0$ imply $1 \geq l_{e,t} \geq \gamma$.

As a short summary of the Ramsey program, if an interior steady-state with $\tau^K > 0$ exists, then equation (29) becomes:

$$1 - \beta R = \frac{F_L - w}{w}(1-\gamma)(\sigma - 1) l_e, \tag{79}$$

and the Straub–Werning condition always holds.[34] To prove the latter, we use the properties of the KPR utility function and the agents' FOCs to express prices in the aggregate budget constraint $c_e + \frac{1}{R} c_u = w l_e$ and in the wedge equation (79). Using the same steps as in

[34]It can also be shown that the no first-best condition is: $0 \leq \frac{K_{FB}}{L_{FB}} l_{FB} - \frac{\beta}{1-\beta} G + \frac{\beta}{1+\beta} w_{FB} (\frac{\gamma}{1-\gamma}(1 - l_{FB} - (1-l_{FB})^{\frac{\sigma}{1+\gamma(\sigma-1)}}) - l_{FB})$, where l_{FB} is the unique root of $l \in \mathbb{R}_+ \mapsto \frac{\gamma}{1-\gamma} w_{FB} (1 - l + (1-l)^{\frac{\sigma}{1+\gamma(\sigma-1)}}) + G - y_{FB} l$.

Section A, we obtain:

$$0 = \frac{\gamma - l_e}{1 - l_e} U(c_e, l_e) + \beta\gamma U(c_u, 0),$$
$$1 - \frac{U(c_e, l_e)}{U(c_u, 0)} \frac{c_u}{c_e} = \left(w_{FB}\gamma \frac{1 - l_e}{c_e} - (1 - \gamma) \right) (\sigma - 1) l_e,$$
$$\frac{U_c(c_e, l_e)}{\mu} = 1 + (F_L/w - 1)(\gamma - l_e)(\sigma - 1). \tag{80}$$

These three equations, together with the resource constraint correspond to the system characterizing the allocation and the multiplier μ of the resource constraint.

Finally, substituting the expressions of R and w, (80) becomes: $\frac{U_c(c_e,l_e)}{\mu} = 1 - \left(1 - \frac{U(c_e,l_e)}{U(c_u,0)} \frac{c_u}{c_e}\right) \frac{1-\gamma/l_e}{1-\gamma}$. Note that $\frac{1-\gamma/l_e}{1-\gamma} \in (0, 1]$ since $l_e \in [\gamma, 1)$ (see above). We deduce that $\frac{U_c(c_e,l_e)}{\mu} \geq \min(1, \frac{U(c_e,l_e)}{U(c_u,0)} \frac{c_u}{c_e}) > 0$. Hence, $\mu > 0$, which concludes the proof.

Numerical illustration. We consider the calibration of Table 9. The preference parameters (σ and γ) are in the same ballpark as to those of Dyrda and Pedroni (2022). The other parameter (β, α, and δ) are set to the standard values of Table 3. In the equilibrium with

Parameters	inverse of IES, σ	consumption share, γ	public spending G
Value	2.0	0.6	0.5

Table 9: Calibration of an economy with a KPR utility function.

binding credit constraints for unemployed agents, this calibration generates the allocation and prices described in Table 10. The taxes and NDG are positive.

Allocation and taxes		
employed agents	consumption c_e	0.644
	labor supply l_e	0.719
unemployed agents	consumption c_u	0.462
taxes	capital tax τ^K	58.982%
	labor tax τ^L	7.582%

Table 10: Allocation in the economy with the calibration of Table 9.

C The GHH Utility Function

C.1 Proof of Proposition 2

The first-best allocation. The Lagrangian associated to the first-best allocation is: $\mathcal{L}^{FB} = \sum_{t=0}^{\infty} \beta^t (\log(c_{u,t}) + \log(c_{e,t} - \chi^{-1} \frac{l_{e,t}^{1+1/\varphi}}{1+1/\varphi}) + \sum_{t=1}^{\infty} \beta^t \mu_t (K_{t-1} + K_{t-1}^{\alpha} l_{e,t}^{1-\alpha} - \delta K_{t-1} - c_{e,t} - c_{u,t} - G_t - K_t)$, together with non-negativity constraints $c_{e,t}, c_{u,t}, l_{e,t} \geq 0$, which are not binding. In that case, it is straightforward to check that the linear independence constraint qualification (LICQ) holds and the optimization yields a maximum (see Sections C.2 and C.3 below). Denoting by $L_{FB} := l_e$ the steady-state labor supply, the FOCs imply, at the steady state, $c_{u,FB} = c_{e,FB} - \chi^{-1} \frac{L_{FB}^{1+1/\varphi}}{1+1/\varphi}$ (computed using the resource constraint (16)) and:

$$\frac{K_{FB}}{L_{FB}} = \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1}\Big)^{\frac{1}{1-\alpha}}, \; L_{FB} = (\chi(1-\alpha))^{\varphi} \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1}\Big)^{\frac{\alpha}{1-\alpha}\varphi}, \tag{81}$$

$$Y_{FB} = (\chi(1-\alpha))^{\varphi} \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1}\Big)^{\frac{\alpha(1+\varphi)}{1-\alpha}}, \; K_{FB} = (\chi(1-\alpha))^{\varphi} \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1}\Big)^{\frac{1+\alpha\varphi}{1-\alpha}}. \tag{82}$$

The decentralization of the first-best allocation. We now analyze the necessary and sufficient conditions for which the first-best allocation can be decentralized. Using the Euler equations (20) and (21) with equality, one finds: $\beta R_{FB} = 1$. Distorting taxes are also null: $\tau^K = \tau^L = 0$, while the government budget constraint (23) implies that the public debt verifies: $B_{FB} = -\frac{\beta}{1-\beta} G < 0$. To implement the first-best allocation, we further need to check that no agent is credit-constrained.

Since $\beta R_{FB} = 1$ and $L_{FB} = l_{e,FB} = (\chi w_{FB})^{\varphi}$, we obtain the same capital-to-labor ratio and labor supply as in (81), the same output and capital as in (82). The wage is:

$$w_{FB} = (1-\alpha) \Big(\frac{\alpha}{\frac{1}{\beta} + \delta - 1}\Big)^{\frac{\alpha}{1-\alpha}}. \tag{83}$$

Furthermore, since agents are unconstrained, Euler equations and budget constraints imply: $R_{FB} a_{u,FB} - a_{e,FB} + \frac{w(\chi w)^{\varphi}}{\varphi+1} = R_{FB} a_{e,FB} - a_{u,FB}$. Using the financial market clearing condition stating that $a_{e,FB} + a_{u,FB} = B_{FB} + K_{FB}$ implies: $2\frac{1-\beta}{\beta}\frac{a_{u,FB}}{Y_{FB}} = \overline{g}_1 - \frac{G}{Y_{FB}}$,with $\overline{g}_1$ defined in (35). The credit constraint $a_{u,FB} \geq 0$ and equation (82) imply the first-best condition $\frac{G}{Y_{FB}} \leq \overline{g}_1$, which concludes the proof of Proposition 2.

C.2 Constraint Qualification

In our problem, even though the objective function is concave, the equality constraints are not linear and the standard Slater (1950) conditions do not apply. However, we can check that the linear independence constraint qualification (LICQ) holds in our problem. This constraint qualification requires the gradients of equality constraints to be linearly independent at the optimum (or equivalently that the gradient is locally surjective). At any date t, two constraints matter for the instruments of date t. These are the constraints at dates t and $t+1$. We can check that their gradient can be written as:

$$\begin{pmatrix} 1 & \varphi(\chi w_t)^\varphi \frac{\tilde{w}_t}{w_t} - (\varphi+1)(\chi w_t)^\varphi & -\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^\varphi}{1+\varphi} \\ -\tilde{r}_{t+1} - 1 & \frac{\beta}{1+\beta}(\chi w_t)^\varphi \tilde{r}_{t+1} - (R_{t+1}-1)\frac{\beta}{1+\beta}\frac{w_t(\chi w_t)^\varphi}{1+\varphi} & 0 \end{pmatrix}, \tag{84}$$

which forms a matrix of rank 2. Indeed, looking at the first and third columns of the matrix in (84) makes it clear that a sufficient condition is $(1+\tilde{r}_{t+1})w_{t-1} \neq 0$. This condition must hold at the optimum, since equation (1) implies $\tilde{r}_{t+1} \geq 0$ and $w_{t-1} > 0$.

C.3 Second-Order Conditions

In the program (17)–(26), we use (23) to substitute for the expression of R_t. We can further use financial market constraint (25) to express B_t as a function of K_t and w_t. The planner's program can be equivalently rewritten as a function of K_t and $W_t = w_t(\chi w_t)^\varphi$:

$$\max_{(K_t,w_t)_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log(W_t) + \log\left(K_{t-1} + F(K_{t-1}, \chi^{\frac{\varphi}{1+\varphi}} W_t^{\frac{\varphi}{1+\varphi}}) - \frac{1+\varphi+\varphi\beta}{(1+\beta)(1+\varphi)} W_t - K_t - G_t\right)\right).$$

The function $(W_t, K_{t-1}) \mapsto F(K_{t-1}, \chi^{\frac{\varphi}{1+\varphi}} W_t^{\frac{\varphi}{1+\varphi}})$ is concave as the composition of concave and increasing functions. We thus deduce that the mapping defined by $(W_t, K_{t-1}, K_t) \mapsto \log(W_t) + \log(K_{t-1} + F(K_{t-1}, \chi^{\frac{\varphi}{1+\varphi}} W_t^{\frac{\varphi}{1+\varphi}}) - \frac{1+\varphi+\varphi\beta}{(1+\beta)(1+\varphi)} W_t - K_t - G_t)$ is concave. Any interior optimum characterized by the FOCs must thus be a maximum.

C.4 Proof of Proposition 3

FOCs Derivations. We first derive the FOCs of the model featuring binding credit constraints. In the GHH case, labor supply of employed agents is $l_{e,t} = (\chi w_t)^\varphi$. Using individual budget constraints and log utility, Euler equations (20) becomes: $a_{e,t} = \frac{\beta}{1+\beta}\frac{w_t(\chi w_t)^\varphi}{1+\varphi} \geq 0$.

The Ramsey program can then be written as:

$$\max_{\{B_t,w_t,R_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(\frac{1}{1+\beta} \frac{w_t(\chi w_t)^{\varphi}}{\varphi+1} \right) + \log(R_t \frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi}) \right), \tag{85}$$

$$\text{s.t.} \quad w_{t+1}(\chi w_{t+1})^{\varphi} > \beta^2 R_{t+1} R_t w_t (\chi w_t)^{\varphi}, \tag{86}$$

$$G + B_{t-1} + (R_t - 1)\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} + w_t(\chi w_t)^{\varphi} = B_t \tag{87}$$

$$+ F(\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} - B_{t-1}, (\chi w_t)^{\varphi}).$$

Note that the Euler inequality for unemployed agents (86) is equivalent at the steady state to $\beta R < 1$, which will always hold in equilibrium (see below).

Defining by convention w_{-1} as $\frac{\beta}{1+\beta}\frac{w_{-1}(\chi w_{-1})^{\varphi}}{1+\varphi} = a_{-1}$ and by $\beta^t \mu_t$ the Lagrange multiplier on (87), the FOCs associated to the program (85)–(87) can be written as (for $t \geq 0$):

$$\mu_t = \beta(1 + F_{K,t+1})\mu_{t+1}, \tag{88}$$

$$1 = R_t \mu_t \frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi}, \tag{89}$$

$$0 = (1+\beta)(\varphi+1)\frac{1}{w_t} + \beta(\chi w_t)^{\varphi}\frac{\beta}{1+\beta}\mu_{t+1}(F_{K,t+1} - R_{t+1} + 1) \tag{90}$$

$$+ \chi\mu_t(\chi w_t)^{\varphi-1}\left(\varphi F_{L,t} - (\varphi+1)w_t\right),$$

We can take advantage of FOCs (88) and (89) to simplify FOC (90) as follows: $\mu_t w_t (\chi w_t)^{\varphi}(1 - (1+\beta)\varphi\frac{\tau_t^L}{1-\tau_t^L}) = (1+\varphi)(1+\beta)$, which is a time-$t$ equation only. The only dynamic FOC is the forward-looking equation (88). We will check that the system is well-defined and does not raise convergence issues.

Note that because of FOC (89), $\mu = 0$ or $R = 0$ is not possible at the steady state. FOCs (88)–(90) and governmental budget constraint (87) become at the steady state, where we denote variables without subscripts:

$$\frac{1}{1+\beta}\mu w(\chi w)^{\varphi} = \varphi + 1 + \mu(\chi w)^{\varphi}\varphi(F_L - w), \tag{91}$$

$$1 = \beta(1 + F_K) \tag{92}$$

$$1 = R\mu\frac{\beta}{1+\beta}\frac{w(\chi w)^{\varphi}}{1+\varphi} \tag{93}$$

$$F(\frac{\beta}{1+\beta}\frac{w(\chi w)^{\varphi}}{1+\varphi} - B, (\chi w)^{\varphi}) = G + (R-1)\frac{\beta}{1+\beta}\frac{w(\chi w)^{\varphi}}{1+\varphi} + w(\chi w)^{\varphi}. \tag{94}$$

We can check that equations (91) and (93) yield $\tau^K = \varphi\frac{1+\beta}{1-\beta}\frac{\tau^L}{1-\tau^L}$ (equation (34)). Finally, the steady-state Ramsey allocation for the interior solution is the solution of the system of two equations in R and w, using (87), and (29): $w = \frac{(F(k_{FB},1)-G)(1+\varphi)+w_{FB}\varphi}{1+2\varphi+\frac{1-\beta}{1+\beta}}$, where $k_{FB} := K_{FB}/L_{FB} = \left(\frac{1/\beta-1+\delta}{\alpha}\right)^{-\frac{1}{1-\alpha}}$ and $w_{FB} = (1-\alpha)k_{FB}^{\alpha}$. Public debt is $B = a_e - K = (\chi w)^{\varphi}\left(\frac{\beta}{1+\beta}\frac{1-\tau^L}{1+\varphi}w_{FB} - k_{FB}\right)$.

The Laffer threshold. Using (34) and (92), as well as the properties of F, the governmental budget constraint (94) implies that τ^L is a solution of $\mathcal{T}(\tau^L) = 0$, where:

$$\mathcal{T} : \tau \in (-\infty, 1) \mapsto \tau - \frac{1}{1-\alpha}\frac{\frac{G}{Y_{FB}}(1-\tau)^{-\varphi} - \overline{g}_1}{1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi} + \frac{\varphi}{1+\varphi}}. \tag{95}$$

The mapping $\tau \mapsto \mathcal{T}(\tau)$ is akin to a Laffer curve. Indeed, we can check that $\mathcal{T}$ is continuously differentiable, strictly concave, with a unique maximum over $(-\infty, 1)$. In consequence, the function $\mathcal{T}$ admits either zero, one, or two solutions. The number of solutions depends on the level of public spending G in (95). When public spending is too high, there is no level of labor tax that makes this public spending sustainable: $T(\tau) < 0$ for all $\tau \in (-\infty, 1)$. When the public spending is sustainable, T typically admits two roots. The smaller root corresponds to a low tax and a high labor supply, while the larger root corresponds to a high tax and a low labor supply. There is a third case that is the limit between sustainability and no sustainability. In this situation, there is a unique tax rate that enables public spending to be financed.

The limit case of the Laffer curve happens when the extremum point of the Laffer curve is the only root of the function. It can be checked that this corresponds to the tax level $\overline{\tau}_{La}^L$ that verifies $\mathcal{T}(\overline{\tau}_{La}^L) = \mathcal{T}'(\overline{\tau}_{La}^L) = 0$ and defined in (37), which is well-defined since $\overline{g}_1 \geq -\frac{1-\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}$. This corresponds to a ratio of public spending $\overline{g}_{La}$ of equation (36). So, any public spending such that $\frac{G}{Y_{FB}} > \overline{g}_{La}$ cannot be financed by any tax system.

Oppositely, when $\frac{G}{Y_{FB}} < \overline{g}_{La}$, two different tax levels enable the government to finance public spending, and the planner will always opt for the lowest tax rate. Indeed, taxes have an unambiguously negative impact on consumption levels, since: $C_e = \frac{1}{1+\beta}(1-\tau^L)^{\varphi+1}\frac{w_{FB}(\chi w_{FB})^{\varphi}}{1+\varphi}$ and $c_u = C_u = (1-(1-\beta)\tau^K)C_e$. So larger taxes decrease consumption and hence individual welfare.

As a conclusion, let us prove that $\overline{g}_{La} \geq \overline{g}_1$ and more precisely the following lemma.

Lemma 1. *We have $\overline{g}_{La} \geq \overline{g}_1$. The equality only holds if $\frac{\varphi}{1-\alpha}\frac{\overline{g}_1}{1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}} = 1$. Otherwise, the inequality is strict.*

Proof. Note that by construction, $\overline{g}_{La} \geq 0$. The result thus holds if $\overline{g}_1 < 0$. We assume that $\overline{g}_1 \geq 0$. Using the definitions of $\overline{g}_{La}$ and $\overline{g}_1$, we have:

$$\frac{\overline{g}_{La} - \overline{g}_1}{\kappa} = (\frac{\varphi}{1+\varphi})^{\varphi} \frac{1-\alpha}{1+\varphi}(1 + \frac{\overline{g}_1}{\kappa})^{1+\varphi} - \frac{\overline{g}_1}{\kappa},$$

with $\kappa = (1-\alpha)(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi} + \frac{\varphi}{1+\varphi}) > 0$. The sign of $\overline{g}_{La} - \overline{g}_1$ can be determined by focusing on the function $s : x \in \mathbb{R}_+ \mapsto \left(\frac{\varphi}{1+\varphi}\right)^{\varphi} \frac{1}{1+\varphi}(1+x)^{1+\varphi} - x$, which is well-defined and continuously differentiable on $\mathbb{R}_+$. We have $s'(x) \geq 0$ iff $\left(\frac{\varphi}{1+\varphi}\right)^{\varphi} (1+x)^{\varphi} \geq 1$ or $x \geq \varphi^{-1}$. The function s thus admits a minimum for $x = \varphi^{-1}$, whose value is: $s(\varphi^{-1}) = \left(\frac{\varphi}{1+\varphi}\right)^{\varphi} \frac{1}{1+\varphi}(\frac{1+\varphi}{\varphi})^{1+\varphi} - \frac{1}{\varphi} = 0$. We deduce that $s(x) \geq 0$ and the equality holds iff $x = \varphi^{-1}$, which concludes the proof. □

The Straub-Werning threshold. The relationship (34) does not provide any upper bound on the capital tax, which diverges when τ^L becomes close to 100%. However, the post-tax interest rate sets an implicit bound on the capital tax. Indeed, the post-tax interest rate must remain positive – otherwise unemployed agents would face negative consumption. The positivity of the post-tax rate is equivalent to the positivity of the Lagrange multiplier μ through FOC (93). Since $x_u = \frac{1-(1-\beta)\tau^K}{1+\beta}\frac{w(\chi w)^{\varphi}}{1+\varphi}$, the capital tax must remain below a threshold

$$\overline{\tau}^K_{SW} := \frac{1}{1-\beta}. \tag{96}$$

When τ^K increases toward $\overline{\tau}^K_{SW}$, the real interest rate tends toward 0, and as w is finite (due to the budget constraint of the government), then μ tends toward $+\infty$. In other words when the capital tax increases toward $\overline{\tau}^K_{SW}$ the planner finds it infinity costly to implement a steady-state optimal allocation. After this threshold, such an equilibrium cannot exist.

This tax threshold implies an upper bound on the labor tax $\overline{\tau}^L_{SW}$ (through equation (34)) and also an upper bound on the level of public spending: $G < \overline{g}_{SW} Y_{FB}$, where $\overline{g}_{SW}$ is defined in (38). This concludes the proof of Proposition 3.

Checking that transfers are zero in a SRE with a positive capital tax. The previous fiscal structure assumed that the planner could not implement positive transfers. We prove here that this actually results from an optimal planner's decision (i.e., the planner would like to implement lump-sum taxes) in any SRE with positive capital tax. A reformulation of the program (85) with transfer generates the necessary and sufficient condition for transfers to be zero is $\frac{1}{x_e} + \frac{1}{x_u} < 2\mu$, where $x_e = \frac{1}{1+\beta}\frac{w(\chi w)^{\varphi}}{\varphi+1}$ and

$x_u = R\frac{\beta}{1+\beta}\frac{w(\chi w)^{\varphi}}{1+\varphi}$. In words, the utility increase of both agents for an marginal increase in transfer (the left hand side) is lower than the marginal cost for the planner (the right hand side). Using (93) stating that $1 = \mu x_u$ and the Euler equation $x_u = \beta R x_e$, the zero-transfer condition becomes $\beta R + 1 < 2$, which is equivalent to $\beta R < 1$ and $\tau^K > 0$. Transfers are zero in any SRE with positive capital tax.

C.5 The $\tau^K = 0$-Equilibrium

We prove here that the steady-state equilibrium featuring $\tau^K = 0$ is always dominated by the equilibrium featuring binding credit constraint and $\tau^K > 0$. We write with the 0-subscript the allocation where $\tau^K = 0$, and with no subscript the allocation where $\tau^K > 0$. The proof is split into three parts: (i) the characterization of the $\tau^K = 0$-equilibrium (Section C.5.1); (ii) when the $\tau_k > 0$-equilibrium exists, i.e., when the Straub-Werning condition holds (Section C.5.2); and (iii) when the $\tau_k > 0$-equilibrium does not exist, i.e., when the Straub-Werning condition does not hold (Section C.5.3).

C.5.1 Characterization of the $\tau^K = 0$-Equilibrium

With the same steps as in Section C.1, we have: $w_0 = (1-\tau^L)w_{FB}$, $K_0 = (1-\tau^L)^{\varphi}K_{FB}$, $Y_0 = (1-\tau^L)^{\varphi}Y_{FB}$. Governmental budget constraint becomes: $B_0 = -\frac{\beta}{1-\beta}G + \frac{\beta}{1-\beta}\tau_0^L(1-\tau_0^L)^{\varphi}w_{FB}(\chi w_{FB})^{\varphi}$. Perfect risk sharing (i.e., $c_{u,0} = c_{e,0} - \frac{1}{\chi}\frac{l_{e,0}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$) and financial market clearing (i.e., $A_0 = K_0 + B_0$) imply after some manipulations:

$$2\frac{a_{u,0}}{Y_0} = \frac{\beta}{1-\beta}(\overline{g}_1 - g_{FB}(1-\tau_0^L)^{-\varphi}) + \left(\frac{1}{1-\beta} + \frac{1}{1+\beta}\frac{1}{\varphi+1}\right)\beta\tau_0^L(1-\alpha), \tag{97}$$

$$2\frac{a_{e,0}}{Y} = 2\frac{a_{u,0}}{Y} + 2\frac{\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}(1-\tau_0^L), \tag{98}$$

meaning that $a_{e,0} \geq a_{u,0}$ for all values of $\tau_0^L \leq 1$. We compute the consumption level $c_{u,0}$ from individual budget constraint:

$$2\frac{c_{u,0}}{Y_{FB}} = (1-\tau_0^L)^{\varphi}\overline{g}_1 - \frac{G}{Y_{FB}} + \frac{2}{1+\beta}\frac{1-\alpha}{\varphi+1}(1-\tau_0^L)^{\varphi} + \frac{\varphi}{\varphi+1}(1-\alpha)\tau_0^L(1-\tau_0^L)^{\varphi}. \tag{99}$$

Computing the derivative of $2\frac{c_{u,0}}{Y_{FB}}$ with respect to the labor tax τ_0^L yields: $\frac{1}{\varphi(1-\tau_0^L)^{\varphi-1}}\frac{\partial}{\partial\tau_0^L}2\frac{c_{u,0}}{Y_{FB}} = -\frac{(1-\beta)\alpha}{1+\beta(\delta-1)} - (1-\alpha)\tau_0^L < 0$, whenever $\tau_0^L \geq 0$. We deduce from the last inequality that $c_{u,0}$ is decreasing with τ_0^L (and hence aggregate welfare since $c_{u,0} = c_{e,0} - \frac{1}{\chi}\frac{l_0^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$). Since

$a_{e,0} \geq a_{u,0}$ for all values of τ_0^L, the value of τ_0^L is chosen as small as possible for credit constraints not to bind and hence such that $a_{u,0} = 0$. From (97), τ_0^L is the solution of:

$$\tau_0^L = \frac{1}{1 + \frac{1-\beta}{1+\beta}\frac{1}{\varphi+1}} \frac{g_{FB}(1-\tau_0^L)^{-\varphi} - \overline{g}_1}{1-\alpha}, \tag{100}$$

which is a Laffer-like curve, as (95), admitting 0, 1 or 2 solutions. Finally, regarding allocation, we have:

$$c_{u,0} = c_{e,0} - \chi^{-1}\frac{l_0^{1+1/\varphi}}{1+1/\varphi} = \frac{1}{1+\beta}\frac{w_0(\chi w_0)^{\varphi}}{1+\varphi}. \tag{101}$$

C.5.2 Case where the $\tau_k > 0$-Equilibrium Exists

The allocations with $\tau^K = 0$ and $\tau^K > 0$ can be written as the outcomes of the same program, with the constraint $\tau^K \geq 0$. Indeed, consider the program:

$$\max_{\{B_t, w_t, R_t\}} \sum_{t=0}^{\infty} \beta^t \left((1+\beta)\log\left(\frac{1}{1+\beta}\frac{w_t(\chi w_t)^{\varphi}}{\varphi+1}\right) + \log(\beta R_t) \right) \tag{102}$$

$$G + B_{t-1} + (R_t - 1)\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} + w_t(\chi w_t)^{\varphi} = B_t \tag{103}$$

$$+ F\left(\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} - B_{t-1}, (\chi w_t)^{\varphi}\right),$$

with $R_t \geq 1 + \tilde{r}_t$, where $\tilde{r}_t = F_{K,t}$ is exogenous. We now show that the previous program has the desired properties.

We start with the case $\tau^K = 0$. Denoting by $\beta^t \mu_t$ the Lagrange multiplier associated to the constraint (103), the maximization with respect to B_t yields: $\mu_t = \beta(1 + F_{K,t+1})\mu_{t+1}$, or at the steady state: $\beta(1 + F_K) = 1$. The constraint (103) implies then at the steady state, using (81)–(82), that the labor tax, denoted $\hat{\tau}_0^l$ verifies: $(1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)\hat{\tau}_0^l = \frac{g_{FB}}{(1-\hat{\tau}_0^l)^{\varphi}} - \overline{g}_1$, which is the same equation as (100) for τ_0^L. Since the planner will also choose the lowest solution, we deduce that $\hat{\tau}_0^l = \tau_0^L$. Consumption levels then mechanically verify equation (101), which proves that the steady-state equilibrium with $\tau^K = 0$ is a steady-state solution of the program (102)–(103) where we impose $\tau_t^K = 0$ at all dates.

We now turn to the unconstrained case ($\tau^K \neq 0$). In that case, the FOCs of the program (102)–(103), with respect to B_t, R_t, and w_t, respectively, are:

$$\mu_t = \mu_{t+1}\beta(1 + F_{K,t}), \tag{104}$$

$$1 = R_t\mu_t \frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi}, \tag{105}$$

$$\begin{aligned} \frac{(1+\beta)(1+\varphi)}{w_t} &= \frac{\mu_t}{w_t}\left((\varphi+1)w_t(\chi w_t)^{\varphi} - \varphi F_{L,t}(\chi w_t)^{\varphi}\right) \\ &+ \frac{\beta\mu_{t+1}}{w_t}(R_{t+1} - 1 - F_{K,t+1})\frac{\beta}{1+\beta}w_t(\chi w_t)^{\varphi}, \end{aligned} \tag{106}$$

which are identical to the FOCs (90)–(89) of the unconstrained case.

We therefore deduce that the allocation with $\tau^K = 0$ is the solution of a constrained program and is hence dominated by the allocation $\tau_k \neq 0$ – whenever the latter exists.[35]

C.5.3 Case where the $\tau_k > 0$-Equilibrium Does Not Exist

We now show that an equilibrium with $\tau^K = 0$ does not exist even when the equilibrium where $\tau^K > 0$ does not exist. Assume now that the solution of (95) does not verify the Straub-Werning condition. We will show that in that case the $\tau_k = 0$-equilibrium does not exist either. To do so, we focus on the limit case when the Straub-Werning condition does not hold, implying that the solution to (95) is $\tau^L_m = \frac{1}{1+(1+\beta)\varphi}$. The argument easily extends to any value $\tau^L \geq \tau^L_m$ (see explanation after equation (108)). Equation (95) implies that it corresponds to a public spending $g_{FB,0}$ verifying:

$$g_{FB,0}(1-\tau^L_m)^{-\varphi} = (1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi} + \frac{\varphi}{1+\varphi}\right)\tau^L_m + \overline{g}_1. \tag{107}$$

To show that the $\tau_k = 0$-equilibrium does not exist, we show that there is no solution to (100), and more precisely that, for all τ^L_0:

$$\tau^L_0 < \frac{g_{FB,0}(1-\tau^L_0)^{-\varphi} - \overline{g}_1}{(1-\alpha)\left(1 + \frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}. \tag{108}$$

The argument we develop would easily extend to any solution τ^L to (95), such that $\tau^L \geq \tau^L_m$. Indeed, these cases would imply public spending levels higher than $g_{FB,0}$. The equilibrium non-existence would then be implied by inequality (108).

To show inequality (108), notice that $\tau_0 \in (-\infty, 1) \mapsto g_{FB,0}(1-\tau^L_0)^{-\varphi} - \overline{g}_1 - (1 -$

[35] Note that the argument could not be applied right away from the initial program formulation because with $\tau_k \neq 0$, the constraint $a_{u,t} = 0$ was binding – which is not present anymore with the modified program (102)–(103).

$\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)\tau_0^L$ is convex admits a global minimum $\tau_{0,\min}^L = (\frac{\varphi g_{FB,0}}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)})^{\frac{1}{\varphi+1}}$. To prove inequality (108), we only need to show that $\Delta > 0$, where

$$\Delta = \frac{(1+\beta)(\varphi+1)}{1+(1+\beta)\varphi}\left(\frac{2(1+(1+\beta)\varphi)}{(1+\beta)((1+\beta)(1+\varphi)+1-\beta)} + \frac{\frac{1+(1+\beta)\varphi}{1+\beta}\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}\right)^{\frac{1}{\varphi+1}} \quad (109)$$
$$- \frac{\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)} - 1,$$

which can be seen as a function of $\tilde{g}_1 = \frac{\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)}$, defined on $(-\frac{2}{(1+\beta)(1+\varphi)+1-\beta}, \infty)$. This function is concave, admits a unique maximum, $\frac{(1+\beta)\varphi}{(1+\beta)(1+\varphi)+1-\beta} > 0$, in $\tilde{g}_1^* = \frac{-2\varphi(1+\beta)}{(1+(1+\beta)\varphi)((1+\beta)(1+\varphi)+1-\beta)}$. Thus, there exist two (mathematical) bounds denoted $\tilde{g}_1^{\text{inf}} < \tilde{g}_1^* < \tilde{g}_1^{\text{sup}}$, such that $\Delta(\tilde{g}_1) > 0$ iff $\tilde{g}_1 \in (\tilde{g}_1^{\text{inf}}, \tilde{g}_1^{\text{sup}})$. The rest of the proof consists in finding two economical bounds on $\tilde{g}_1$, denoted by $\tilde{g}_1^{\min}$ and $\tilde{g}_1^{\max}$ and to prove that $\Delta(\tilde{g}_1^{\min}) > 0$ and $\Delta(\tilde{g}_1^{\max}) > 0$. We can then deduce from the properties of the function Δ that $\Delta(\tilde{g}_1) > 0$ for all economically acceptable $\tilde{g}_1$, which concludes the proof.

Lower bound on $\tilde{g}_1$. The definition (35) of $\overline{g}_1 = \frac{1-\beta}{\beta}\frac{\alpha}{1/\beta+\delta-1} - \frac{1-\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}$ readily implies: $\frac{\overline{g}_1}{(1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}\right)} \geq -\frac{1-\beta}{(1+\beta)(1+\varphi)+1-\beta} = \tilde{g}_1^{\min}$, or from (109): $\Delta(\tilde{g}_1^{\min}) \geq \frac{(1+\beta)(1+\varphi)}{(1+\beta)(1+\varphi)+1-\beta}\left((1+\frac{1}{1+(1+\beta)\varphi})^{\frac{\varphi}{\varphi+1}} - 1\right) > 0$, where the second inequality comes from $\beta \in (0,1)$ and $\varphi > 0$.

Upper bound on $\tilde{g}_1$. The upper bound on $\tilde{g}_1$ is less straightforward. Equation (107) – seen as an equation in τ_m^L for a given $g_{FB,0}$ – admits one or two roots (since by construction the no-root case is excluded). To guarantee that the smallest solution is chosen, the derivative of the $\tau \mapsto (1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)\tau + \overline{g}_1 - g_{FB,0}(1-\tau)^{-\varphi}$ must be positive in τ_m^L (the function being concave, it has to intercept 0 before it reaches its maximum). Or equivalently: $\varphi g_{FB,0}(1-\tau_m^L)^{-\varphi-1} \leq (1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)$. Using (107), we obtain that this condition is equivalent to: $\frac{\overline{g}_1}{(1-\alpha)(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi})} \leq \frac{2\beta}{(1+\beta)(1+\varphi)+1-\beta} = \tilde{g}_1^{\max}$. From (109), we obtain, after some manipulations:

$$\frac{\Delta(\tilde{g}_1^{\max})}{\tau_m^L} \geq (1+\beta)(1+\varphi)\left(\left(1+\frac{\varphi(1+\beta)}{1+(1+\varphi(1+\beta))}\right)^{\frac{1}{\varphi+1}} - 1\right) - \beta\frac{\varphi(1+\beta)}{(1+(1+\varphi(1+\beta)))},$$

whose left-handside can be seen as a function of $\frac{\varphi(1+\beta)}{1+(1+\varphi(1+\beta))}$ (that lies in $(0,1)$). We denote: $\tilde{\Delta} : x \in (0,1) \mapsto (1+\beta)(\varphi+1)\left((1+x)^{\frac{1}{\varphi+1}} - 1\right) - \beta x$. Using a second-order Taylor development, we have for $x \in (0,1)$: $\frac{\tilde{\Delta}(x)}{x} \geq 1 - \frac{\varphi}{\varphi+1}\frac{1+\beta}{2}x > 0$, where the second inequality comes from $x < 1$, $\beta < 1$, and $\varphi > 0$.

C.6 A Non-Interior Steady-State Equilibrium

When (95) admits a solution that does not verify the Straub-Werning condition, FOC (88) holds and FOCs (90) and (89) can also be written as:

$$\left(1 - (1 + \varphi(1 + \beta))\tau_t^L\right)(1 - \tau_t^L)^{\varphi}\mu_t\tilde{w}_t(\chi\tilde{w}_t)^{\varphi} = (1 + \beta)(1 + \varphi), \tag{110}$$

$$(1 + (1 - \tau_t^K)F_{K,t})\mu_t(1 - \tau_{t-1}^L)^{\varphi+1}\tilde{w}_{t-1}(\chi\tilde{w}_{t-1})^{\varphi} = \frac{(1 + \beta)(1 + \varphi)}{\beta}. \tag{111}$$

Equation (110) implies that for all t: $\tau_t^L \leq \frac{1}{1+\varphi(1+\beta)}$ and $\tau^L = \lim_{t\to\infty}\tau_t^L \leq \frac{1}{1+\varphi(1+\beta)}$. From (110), there are possibly non-interior steady states, featuring $\lim_t \mu_t = \infty$ or $\lim_t \tilde{w}_t = \infty$.

First case: $\lim w_t = w^* < \infty$**.**

- The case $w^* = 0$ is not possible. Otherwise there are no resources to pay G.
- Assume that $\lim \mu_t = \infty$, then equation (110) implies $\lim_t \tau_t^L = (1 + \varphi(1 + \beta))^{-1}$. Equation (111) then yields $\lim_t(1 + (1 - \tau_t^K)F_{K,t}) = \lim_t R_t = 0$.

Second case: $\lim_t w_t = \infty$**.** We thus have $\lim_t \tilde{w}_t = \infty$. Using factor price definitions: $\chi\tilde{w}_t = \left(\frac{\chi(1-\alpha)}{(1-\tau_t^L)^{\alpha\varphi}}\right)^{\frac{1}{1+\varphi\alpha}} K_{t-1}^{\frac{\alpha}{1+\varphi\alpha}}$ yields $\lim_t K_t = \infty$ and $\lim_t \frac{K_{t-1}}{(\chi w_t)^{\varphi}} = \infty$. We deduce $\lim_t F_{K,t} = -\delta$, as well as $\lim_t \mu_t = \infty$, $\lim_t \tau_t^L = (1 + \varphi(1 + \beta))^{-1}$, and $\lim_t R_t = 0$.

These two non-stationary equilibria feature $\lim_t \mu_t = \infty$ and $\lim_t R_t = 0$.

C.7 Characterization of Positive Public Debt

We prove here Result 1. The financial market clearing condition implies using the expression of $a_{e,t}$ (see Section C.4) and the definition of w: $B = (\chi w)^{\varphi}\left(\frac{\beta}{1+\beta}\frac{1-\tau^L}{1+\varphi}F_L - \frac{K}{L}\right)$, which is positive iff: $\frac{\beta}{1+\beta}\frac{1-\tau^L}{1+\varphi} > \frac{1}{F_L}\frac{K}{L}$. Using the definitions of F and $\overline{g}_1$, we can simplify $\frac{1}{F_L}\frac{K}{L}$ and obtain that $B > 0$ iff: $\tau^L < -\frac{1+\varphi}{1-\alpha}\frac{1+\beta}{1-\beta}\overline{g}_1$. Using (95), we get the equivalent condition $g_{FB}(1 - \tau^L)^{-\varphi} < \overline{g}_{\text{pos}}$ the expression of $\overline{g}_{\text{pos}}$ being given in (39).

C.8 Straub-Werning Condition with an IES Different From 1

We derive the conditions for $\mu > 0$, considering a GHH utility function with an IES different from 1. Following the same steps as in Section C.4, we find that the multiplier on the budget of the state is: $\mu = u'(c_u)\frac{\sigma(\beta R+\beta(\beta R)^{\frac{1}{\sigma}})}{\sigma(1+\beta(\beta R)^{\frac{1}{\sigma}})-(1-\beta R)}$. The condition for $\mu > 0$ is $(\beta R)^{\frac{1}{\sigma}} + \frac{1}{\sigma}R > \frac{1}{\beta}\frac{1-\sigma}{\sigma}$. When $\sigma > 1$, the condition is $R > 0$ (which is equivalent to $g < \bar{g}_{SW}$ when $\sigma = 1$). When $\sigma < 1$, the condition is $R > \bar{R}$, where $\bar{R}$ is the unique solution of $(\beta\bar{R})^{\frac{1}{\sigma}} + \frac{1}{\sigma}\bar{R} = \frac{1}{\beta}\frac{1-\sigma}{\sigma}$ (as the left-hand-side is increasing).

We compute the steady-state value of R with the governmental budget constraint and it is found as the solution of:

$$(1-g)\left(\frac{K_{FB}}{L_{FB}}\right)^{\alpha} - \delta\frac{K_{FB}}{L_{FB}} = \frac{w_{FB}\varphi}{1+\varphi}\frac{\sigma(1+\varphi)\beta R + (R+\varphi)(1-\sigma-\beta R)}{(1-\beta R+\sigma\varphi)\beta R + \varphi(1-\sigma-\beta R)}$$

where K_{FB}/L_{FB} is defined in (81), w_{FB} in (83), and $g = G/Y$ is the steady-state value of public spending over GDP. The condition $R > \bar{R}$ implicitly implies an upper bound on g. We can check numerically that the Straub-Werning condition can be satisfied for $\sigma < 1$ if g is low enough (thus R high enough and the capital tax small enough).

From these results, we conclude that the Straub-Werning condition ($\mu > 0$) can be satisfied for values of the IES different from 1.

D Simple Model Dynamics After a Period-0 Shock

D.1 Model Linearization and Proof of Proposition 4

Defining $\theta = \frac{1}{1+\varphi}\frac{\beta}{1+\beta}$, FOCs (90) and (88) and government budget constraint (87) become:

$$\mu_t = \beta(1 + \alpha K_t^{\alpha-1}\chi^{(1-\alpha)\varphi}w_{t+1}^{(1-\alpha)\varphi} - \delta)\mu_{t+1}, \tag{112}$$

$$0 = 1 - \mu_t w_t(\chi w_t)^{\varphi}(1-\theta) + \frac{\varphi}{1+\varphi}\mu_t(1-\alpha)K_{t-1}^{\alpha}(\chi w_t)^{\varphi(1-\alpha)}, \tag{113}$$

$$K_{t-1}^{\alpha}(\chi w_t)^{\varphi(1-\alpha)} = G_t + K_t - (1-\delta)K_{t-1} + \frac{1}{\mu_t} + (1-\theta)w_t(\chi w_t)^{\varphi}. \tag{114}$$

We deduce R_t from $1 = R_t\mu_t\theta w_{t-1}(\chi w_{t-1})^{\varphi}$ (i.e., FOC (89)) and B_t from $B_t = \theta w_t(\chi w_t)^{\varphi} - K_t$ (i.e., financial market clearing). We denote by a hat the proportional deviation to the steady-state value. The linearization of equations (112)–(114) yields:

$$\widehat{\mu}_t = E_t\widehat{\mu}_{t+1} + (1-\beta(1-\delta))((\alpha-1)\widehat{K}_t + (1-\alpha)\varphi E_t\widehat{w}_{t+1}), \tag{115}$$

$$0 = -\alpha\widehat{K}_{t-1} + (A-1)\widehat{\mu}_t + ((\varphi+1)(A-1) + 1 + \varphi\alpha)\widehat{w}_t, \tag{116}$$

$$0 = \frac{G}{Y}\widehat{G}_t + \frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\left(\widehat{K}_t - \beta^{-1}\widehat{K}_{t-1}\right) - (A-1)(1-\alpha)\varphi\left(\frac{\widehat{\mu}_t}{1+\varphi} - \widehat{w}_t\right), \tag{117}$$

where τ^L is defined in (95) and $A := (1 + \frac{1}{\varphi(1+\beta)})(1 - \tau^L) > 1$. The inequality $A > 1$ comes from the Straub-Werning condition.

In the remainder, we will focus on full capital depreciation: $\delta = 1$.

Dynamic system. In that case, we obtain from (115)–(117):

$$E_t\left[\widehat{\mu}_{t+1}\right] = r_\mu \widehat{\mu}_t + t_\mu \widehat{K}_t, \tag{118}$$

$$\widehat{K}_t = r_K \widehat{\mu}_t + t_K \widehat{K}_{t-1} + s_K \widehat{G}_t, \tag{119}$$

where we have defined:

$$r_\mu = \frac{(1+\varphi)(A-1)+1+\alpha\varphi}{(1+\alpha\varphi)A},\ t_\mu = (1-\alpha)\frac{(1+\varphi)(A-1)+1}{(1+\alpha\varphi)A},\ t_K = \frac{1}{\beta}\frac{1}{r_\mu}, \tag{120}$$

$$r_K = \frac{1-\alpha}{\alpha\beta}(A-1)\frac{\varphi}{1+\varphi}\left(1 + \frac{(1+\varphi)(A-1)}{(1+\varphi)(A-1)+1+\varphi\alpha}\right),\ s_K = -\frac{G}{\alpha\beta Y}. \tag{121}$$

Since $A > 1$, it can be checked that the coefficients t_K, r_K, t_μ are positive, while $r_\mu > 1$ and $s_K < 0$. Note that all these coefficients are defined at the steady-state and are independent of the values $\widehat{G}_0, \rho_G$ defining the dynamics of the shock $\widehat{G}_t$.

Deriving a simplified dynamic system. Using an identification method, we look for coefficients ρ_K, σ_K, ρ_μ, σ_μ, such that, for $t > 1$:

$$\widehat{K}_t = \rho_K \widehat{K}_{t-1} + \sigma_K \widehat{G}_t \tag{122}$$

$$\widehat{\mu}_t = \rho_\mu \widehat{K}_{t-1} + \sigma_\mu \widehat{G}_t. \tag{123}$$

Combining (118)–(119) yields: $E_t\widehat{K}_{t+1} - (t_K + r_\mu + r_K t_\mu)\widehat{K}_t + r_\mu t_K \widehat{K}_{t-1} = (s_K\rho_G - r_\mu s_K)\widehat{G}_t$. Using (122), we obtain that ρ_K must solve the following equation: $\rho_K^2 - (t_K + r_\mu + r_K t_\mu)\rho_K + r_\mu t_K = 0$, whose discriminant is: $D = (t_K + r_\mu + r_K t_\mu)^2 - 4 r_\mu t_K$. Since $t_K, r_\mu, r_K, t_\mu \geq 0$, we have $D \geq (t_K + r_\mu)^2 - 4 r_\mu t_K = (t_K - r_\mu)^2 > 0$, where the strict inequality comes from $t_K = \frac{1}{\beta r_\mu} > 0$. The equation thus admits two distinct roots, which are:

$$\rho_{K,1} = \frac{t_K + r_\mu + r_K t_\mu + \sqrt{D}}{2} \text{ and } \rho_{K,2} = \frac{t_K + r_\mu + r_K t_\mu - \sqrt{D}}{2}. \tag{124}$$

Since $(t_K + r_\mu + r_K t_\mu)^2 > D > 0$, we deduce that $0 < \rho_{K,2} < \rho_{K,1}$.

Proof of Proposition 4. Let us now prove that condition (42) is a necessary and sufficient condition for equilibrium stability. Since $0 < \rho_{K,2} < \rho_{K,1}$ and $\rho_{K,2}\rho_{K,1} = \beta^{-1} > 1$, we must have $\rho_{K,1} > 1$, which imposes that $\rho_K = \rho_{K,2}$. The Blanchard-Kahn condition for the system stability requires $\rho_{K,2} < 1$. Note that in the limit case when the equilibrium does not exist (i.e., $\tau^K = \overline{\tau}^K_{SW} = \frac{1}{1-\beta}$), and which corresponds to $A = 1$, it is straightforward to check that $\rho_{K,2} = 1$ and that the dynamic system is not stable. The condition $\rho_{K,2} < 1$ is equivalent to $J := t_K + r_\mu + r_K t_\mu - r_\mu t_K - 1 > 0$. Using equations (120)–(121), we can show that $J = J_0 \times P(A-1)$, where $J_0 = \frac{\varphi(1-\alpha)(A-1)}{\beta(1+\alpha\varphi)A((1+\varphi)(A-1)+1+\varphi\alpha)} > 0$ since $A > 1$ and P is a quadratic polynomial in $A-1$:

$$
\begin{aligned}
P(A-1) = &\frac{1+\alpha\varphi}{1+\varphi}(-(1-\beta)(1+\varphi) + \frac{1-\alpha}{\alpha}) + (A-1)^2\frac{1-\alpha}{\alpha}2(1+\varphi) \\
&+ (A-1)\left(-(1-\beta)(1+\varphi) + \frac{1-\alpha}{\alpha} + 2(1+\alpha\varphi)\frac{1-\alpha}{\alpha}\right).
\end{aligned}
$$

A necessary condition for $P(A-1) > 0$ for all $A > 1$ is $P(0) \geq 0$. However, $P(0) \geq 0 \Rightarrow P'(0) > 0$. So, since $P''(0) \geq 0$, $P(0) \geq 0$ is a necessary and sufficient condition for $P(A-1) > 0$ for $A > 1$. The condition $P(0) \geq 0$ is equivalent to (42), which concludes the proof.. Note that a sufficient condition for stability is $\overline{g}_1 < 0$ since it implies (42).

D.2 Characterizing the Dynamics of Capital and Public Debt and Proof of Proposition 5

Characterization of the system (122)–(123). We deduce from (118)–(119) that $(r_\mu - \rho_K)\rho_\mu = -t_\mu \rho_K$. Since $r_\mu > 1$, $t_\mu > 0$, and $\rho_K \in (0,1)$, we deduce that $\rho_\mu < 0$. Regarding parameters σ_K and σ_μ, we have from (118)–(119):

$$\sigma_K = r_K \sigma_\mu + s_K, \tag{125}$$

$$r_\mu \sigma_\mu = (\rho_\mu - t_\mu)\sigma_K + \sigma_\mu \rho_G. \tag{126}$$

Equation (126) implies $(r_\mu - \rho_G)\sigma_\mu = (\rho_\mu - t_\mu)\sigma_K$. Using $r_\mu > 1 > \rho_G$ and the definition of ρ_μ implying that $\rho_\mu - t_\mu = r_\mu \rho_\mu / \rho_K < 0$, we deduce that σ_μ and σ_K have opposite signs. Using $r_K > 0$ and $s_K < 0$ in equation (125), we deduce that $\sigma_\mu > 0 > \sigma_K$.

The role of the shock persistence ρ_G. Combining (125) and (126) yields: $(r_\mu + (t_\mu - \rho_\mu)r_K)\sigma_\mu = (\rho_\mu - t_\mu)s_K + \sigma_\mu \rho_G$, or using the implicit function theorem: $(r_\mu - \rho_G + (t_\mu - \rho_\mu)r_K)\frac{\partial \sigma_\mu}{\partial \rho_G} = \sigma_\mu$, since only σ_μ (and σ_K) depend on ρ_G. Since $r_\mu > 1 > \rho_G$, and $\sigma_\mu, t_\mu, r_K > 0 > \rho_\mu$, we deduce using the previous equation and (125) that: $\frac{\partial \sigma_\mu}{\partial \rho_G} > 0$ and $\frac{\partial \sigma_K}{\partial \rho_G} > 0$. The previous derivative, and equation (123), imply $\widehat{\mu}_0 = \sigma_\mu \widehat{G}_0$, which implies that for the same initial shock $\widehat{G}_0$, $\frac{\partial\widehat{\mu}_0}{\partial\rho_G}\Big|_{\widehat{G}_0} > 0$. Then, from (116), we have: $\widehat{w}_0 = -\frac{A-1}{((\varphi+1)(A-1)+1+\varphi\alpha}\widehat{\mu}_0$, which implies $\frac{\partial\widehat{w}_0}{\partial\rho_G}\Big|_{\widehat{G}_0} < 0$. Finally, from $\frac{\partial\sigma_K}{\partial\rho_G} > 0$, we deduce $\frac{\partial\widehat{K}_0}{\partial\rho_G} < 0$.

Dynamic of the capital stock. By induction, (40) and (122) imply: $\widehat{G}_t = \rho_G^t\widehat{G}$ and $\widehat{K}_t = \sigma_K\phi(t)\widehat{G}_0$, with $\phi(t) = \frac{\rho_K^{t+1}-\rho_G^{t+1}}{\rho_K-\rho_G}$ if $\rho_K \neq \rho_G$, or $(t+1)\rho_G^t$ if $\rho_K = \rho_G$. We have $\phi(0) = 1$, $\phi(\infty) = 0$. Moreover, $\phi'(t_m) = 0$ iff $t_m + 1 = \frac{\ln(-\ln(\rho_K))-\ln(-\ln(\rho_G))}{\ln(\rho_G)-\ln(\rho_K)} > 0$ if $\rho_K \neq \rho_G$ or $t_m + 1 = -\frac{1}{\ln(\rho_G)} > 0$ if $\rho_K = \rho_G$. It is direct to check that $\phi'(t) > 0$ iff $t < t_m$. The capital response is procyclical (it has the sign of $\widehat{G}_0$). When $\widehat{G}_0 > 0$, capital increases until date t_m before decreasing and converging back to its steady-state value.

We now investigate the impact of ρ_G on t_m. Defining $r_G := -\ln(\rho_G)$ and $r_K := -\ln(\rho_K)$, we obtain: $\frac{\partial t_m}{\partial r_G} = \frac{\frac{r_G-r_K}{r_G}-(\ln(r_G)-\ln(r_K))}{(r_G-r_K)^2}$ if $\rho_K \neq \rho_G$. By the Taylor-Lagrange theorem, there exists $r \in (r_K, r_G)$, such that: $\ln(r_K) - \ln(r_G) = \frac{r_K-r_G}{r_G} - \frac{(r_K-r_G)^2}{2r^2}$, from which we deduce: $\frac{\partial t_m}{\partial r_G} = -\frac{1}{2r^2} < 0$ if $\rho_K \neq \rho_G$ or $\frac{\partial t_m}{\partial r_G} = --\frac{1}{r_G^2} < 0$ if $\rho_K = \rho_G$. So t_m decreases with r_G and increases with ρ_G: the more persistent ρ_G, the longer the impact of capital dynamics.

Dynamics of public debt. Regarding public debt, the financial market clearing implies that $B_t = \frac{\beta}{1+\beta}\frac{\chi^\varphi}{1+\varphi}w_t^{1+\varphi} - K_t$. Defining $\alpha_B := \frac{1}{B}\frac{\beta}{1+\beta}\frac{\chi^\varphi}{1+\varphi}w^{1+\varphi}$, we have: $\widehat{B}_t = \alpha_B\widehat{w}_t - (\alpha_B - 1)\widehat{K}_t$. Using equations (88), (122), and (123), one finds $\widehat{B}_t = \Theta^K\widehat{G}_0\rho_K^t - \Theta^G\widehat{G}_0\rho_G^t$, with:

$$\Theta^K := \left(\alpha_B\frac{\alpha-(A-1)\,\rho_\mu}{(\varphi+1)(A-1)+1+\varphi\alpha} - (\alpha_B-1)\,\rho_K\right)\frac{\sigma_K}{\rho_K-\rho_G}, \tag{127}$$

$$\begin{aligned}\Theta^G := &\left(\alpha_B\frac{\alpha-(A-1)\,\rho_\mu}{(\varphi+1)(A-1)+1+\varphi\alpha} - (\alpha_B-1)\,\rho_K\right)\frac{\sigma_K}{\rho_K-\rho_G} \\ &+ \alpha_B\frac{A-1}{(\varphi+1)(A-1)+1+\varphi\alpha}\sigma_\mu + (\alpha_B-1)\,\sigma_K.\end{aligned} \tag{128}$$

Proof of Proposition 5. At impact ($t = 0$), we have:

$$B\hat{B}_0 = -\left(\frac{\beta}{1+\beta}\chi^\varphi w^{1+\varphi}\frac{A-1}{(\varphi+1)(A-1)+1+\varphi\alpha}\sigma_\mu(\rho_G) + \sigma_K(\rho_G)K\right)\hat{G}_0(\rho_G), \tag{129}$$

where we have explicitly noted the dependence on ρ_G. Recall that $\frac{\partial\sigma_\mu}{\partial\rho_G} > 0$, $\frac{\partial\sigma_K}{\partial\rho_G} > 0$, and since the $N\hat{P}V_0$ is fixed and $\hat{G}_0$ endogenous, $\frac{\partial\widehat{G}_0}{\partial\rho_G}\Big|_{N\hat{P}V} < 0$.

As a consequence, if the public debt is positive at the steady state ($B > 0$ equivalent to $\bar{g}_1 < 0$ – see Section C.7), then for a positive exogenous initial shock, $\hat{G}_0 > 0$, $\frac{\partial\sigma_\mu}{\partial\rho_G} > 0$, $\frac{\partial\sigma_K}{\partial\rho_G} > 0$ imply $\frac{\partial\widehat{B}_0}{\partial\rho_G} < 0$. The higher the shock persistence, the greater the variation of public debt at impact decreases: $\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{\hat{G}_0} < 0$.

In the case of a constant $N\hat{P}V_0$, we have: $B\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{N\hat{P}V_0} = \left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{\hat{G}_0} + \frac{B\widehat{B}_0}{\hat{G}_0(\rho_G)}\left.\frac{\partial \widehat{G}_0}{\partial \rho_G}\right|_{N\hat{P}V_0}$. If in addition to $B > 0$, we also have $\widehat{B}_0 > 0$, we deduce since $\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{\hat{G}_0} < 0$ and $\left.\frac{\partial \widehat{G}_0}{\partial \rho_G}\right|_{N\hat{P}V} < 0$: $B\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{N\hat{P}V_0} < 0$.

D.3 Numerical example

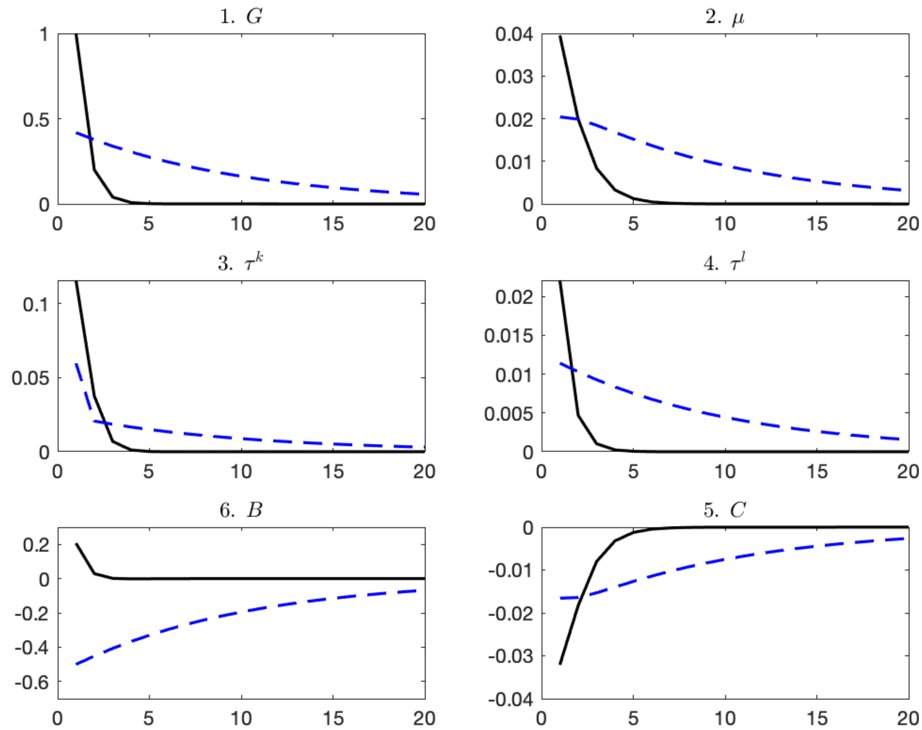

Figure 4: Examples of the dynamics of fiscal variables for a shock with the same net present value and persistences $\rho_G = 0.3$ (black line) and $\rho_G = 0.9$ (blue dashed line). See the text for details.

Figure 4 plots the dynamics of the economy and of the instruments of the planner for two shocks with the same NPV but different persistences, the initial size of the shock $\hat{G}_0$ being adjusted. The parameters are $\alpha = 0.3, \beta = 0.7, \varphi = 0.3, \delta = 1, G = 0.01, \chi = 1$, and one can check that $\bar{g}_1 Y_{FB} < G$, $G \leq \bar{g}_{SW} Y_{FB}$, and $G < \bar{g}_{La} Y_{FB}$. This economy has an equilibrium capital tax of 6%, a labor tax of 3%, and a (small) positive public debt. The low-persistence economy with $\rho_G = 0.2$ corresponds to the black solid line, while the high-persistence economy with $\rho_G = 0.9$ corresponds to the blue dashed line.

Panel 1 plots the increase in public spending. For the increase to be the same in NPV, it increases by 1% on impact in the case of low persistence and by 0.44% in the case of high persistence. Panel 2 plots the social value of public liquidity, μ (i.e., the Lagrange multiplier on the government budget constraint). When the persistence is low, the increase is higher on impact but much less persistent compared to the high-persistence case. Panel 3 plots the capital tax, and panel 4 the labor tax. When the persistence is low, both capital and labor taxes increase more on impact but are much less persistent. Capital tax increases by one order of magnitude more than the labor tax on impact, to front-load the adjustment, because period-0 capital taxes are not distorting (see Farhi, 2010 for a discussion of a similar result with complete insurance markets). However, to avoid reducing the resources of credit-constrained agents, the planner does not fully front-load the adjustment and the labor tax is used on the whole transition. Labor taxes barely increase in both economies. Consequently, there is a long-lasting increase in both capital and labor taxes when the persistence is high. Therefore, any further increase in taxes would be very costly. This creates a strong incentive not to increase public debt, to avoid a higher interest repayment and hence higher taxes. As can be seen in panel 5, public debt increases in the low-persistence economy whereas it decreases in the high-persistence economy. Finally, panel 6 plots aggregate consumption, which falls in both cases, much more so when the persistence is low, but it returns much faster to its steady-state value.

E Proof of Proposition 6

Agent A deterministically transits from employment to unemployment, while agent B is endogenously hand-to-mouth (but works with productivity y^B).

E.1 First-best

The planner chooses $(c^X_{e,t}, l^X_{e,t}, c^X_{u,t})_{X=A,B}, R_t, w_t, B_t$ to maximize the aggregate welfare (45) subject to the resource constraint:

$$\Omega^A c^A_{e,t} + \Omega^B c^B_{e,t} + \Omega^A c^A_{u,t} + G_t + K_t = K_{t-1} + F(K_{t-1}, \sum_X \Omega^X y^X l^X_{e,t}). \tag{130}$$

This yields the following FOCs – with $\beta^t \mu_t$ being the Lagrange multiplier on (130) and $x^X_{e,t} := c^X_{e,t} - \chi^{-1} l^{X,1/\varphi}_{e,t}$. First, $\chi^{-1} l^{X,1/\varphi}_{e,t} = y^X F_{L,t}$ for labor ($X = A, B$), $\mu_t = \frac{\omega^A}{\Omega^A} u'(x^A_{e,t}) = \frac{\omega^B}{\Omega^B} u'(x^B_{e,t})$ and $c^A_{u,t} = x^A_{e,t}$ for consumption and $\mu_t = \beta \mu_{t+1}(1 + F_{K,t+1})$ for public debt. The last equation implies that at the steady–state the capital-to-labor ratio and the wage rate are the same as in the homogeneous economy (equations (81) and (83)). The labor supply is defined by $L_{FB} = \sum_{X=A,B} \Omega_X l_e^{X,1/\varphi}$ and $l_{e,FB}^X = \chi^\varphi y^{X,\varphi} w_{FB}^\varphi$ $(X = A, B)$. Allocations are given by: $c_{u,FB}^A = c_{e,FB}^A - \frac{\varphi\chi^\varphi}{1+\varphi} y^{A,\varphi+1} w_{FB}^{\varphi+1}$ and $c_{e,FB}^B = \frac{\varphi\chi^\varphi}{1+\varphi} y^{B,\varphi+1} w_{FB}^{\varphi+1} + \frac{\omega^B}{\Omega^B}\frac{\Omega^A}{\omega^A}(c_{e,FB}^A - \frac{\varphi\chi^\varphi}{1+\varphi} y^{A,\varphi+1} w_{FB}^{\varphi+1})$ where the consumption level $c_{e,FB}^A$ is determined by the resource constraint (130) at the steady state.

The first-best allocation cannot be decentralized in general. Indeed, we only have two instruments for the decentralization (a_e^A and a_u^A), while have three constraints: the market clearing condition, and the two first-best FOCs: $\frac{\omega^A}{\Omega^A} u'(x_{e,t}^A) = \frac{\omega^B}{\Omega^B} u'(x_{e,t}^B)$ and $c_{u,t}^A = x_{e,t}^A$.

E.2 Full risk-sharing

We now turn to the case where there is no tax $\tau^L = \tau^K = 0$. The labor supplies and consumption levels verify: $l_{e,FRS}^{X,1/\varphi} = \chi w_{FB} y^X$ and $c_{u,FRS}^A = x_{e,FRS}^A$. The latter with the market clearing condition $\Omega_A(a_{e,FRS}^A + a_{u,FRS}^A) = B_{FB} + K_{FB}$ gives asset holdings: $2a_{e,FRS}^A = \frac{B_{FB}+K_{FB}}{\Omega_A} + \frac{\beta}{1+\beta}\frac{\chi^\varphi}{1+\varphi} y^{A,\varphi+1} w_{FB}^{\varphi+1}$ and $2a_{u,FRS}^A = \frac{B_{FB}+K_{FB}}{\Omega_A} - \frac{\beta}{1+\beta}\frac{\chi^\varphi}{1+\varphi} y^{A,\varphi+1} w_{FB}^{\varphi+1}$. The public debt level is imposed by the governmental budget constraint: $B_{FB} = -\beta G/(1-\beta)$. Note that given this allocation, the budget constraints and resource constrain hold.

E.3 The no-capital tax equilibrium

In the absence of a capital tax, there is a perfect risk-sharing between employed and unemployed agents of type A. We thus have at the steady state: $\beta R_{FB} = 1$, $l_{e,\tau^L}^X = (\chi w_{FB} y^X)^\varphi (1-\tau^L)^\varphi$ and $x_{e,\tau^L}^A = c_{u,\tau^L}^A$ (where we denote by the subscript τ^L the variables in this equilibrium).

The governmental budget constraint implies $G + (R_{FB} - 1)B_{\tau^L} = \tau^L w_{FB} L_{\tau^L}$ or using $L_{\tau^L} = (\chi w_{FB})^\varphi (1-\tau^L)^\varphi (\Omega^A (y^A)^{\varphi+1} + \Omega^B (y^B)^{\varphi+1})$, we have:

$$\left(\frac{1}{\beta} - 1\right) B_{\tau^L} = -G + \tau^L (1-\tau^L)^\varphi w_{FB} (\chi w_{FB})^\varphi (\Omega^A (y^A)^{\varphi+1} + \Omega^B (y^B)^{\varphi+1}).$$

The capital market clearing condition gives $\Omega^A(a_{e,\tau^L}^A + a_{u,\tau^L}^A) = B_{\tau^L} + (1-\tau^L)^\varphi (\Omega^A (y^A)^{\varphi+1} + \Omega^B (y^B)^{\varphi+1}) K_{FB}$, while the equation $x_{e,\tau^L}^A = c_{u,\tau^L}^A$ yields:

$$(1+\beta^{-1})(a_{e,\tau^L} - a_{u,\tau^L}) = \frac{\chi^\varphi}{1+\varphi} (w_{FB} y^A)^{\varphi+1} (1-\tau^L)^{\varphi+1}.$$

We deduce from the last two equations the asset holdings and finally the consumption levels from individual budget constraints.

E.4 Case with binding credit constraints

We assume that $\tau_t^K > 0$. The type B agents are credit constrained. The Ramsey program consists for the planner to choose $((c_{e,t}^X, l_{e,t}^X, c_{u,t}^X)_{X=A,B}, a_e, R_t, w_t, B_t)_{t\geq 0}$ to maximize the social welfare (45) subject to the Euler equation of type-A agents $u'(x_{e,t}^A) = \beta R_{t+1} u'(c_{u,t+1}^A)$, the labor supply FOCs $l_{e,t}^X = (\chi w_t y^X)^\varphi$, the individual budget constraints: $c_{e,t}^A + a_{e,t}^A = w_t y^A l_{e,t}^A$, $c_{u,t}^A = R_t a_{e,t-1}^A$, and $c_{e,t}^B = w_t y^B l_{e,t}^B$, as well as the governmental budget constraint:

$$G_t + B_{t-1} + (R_t - 1)\Omega^A a_{e,t-1}^A + w_t(\Omega^A y^A l_{e,t}^A + \Omega^B y^B l_{e,t}^B) = F(\Omega^A a_{e,t-1}^A - B_{t-1}, \Omega^A l_{e,t}^A + \Omega^B l_{e,t}^B) + B_t$$

We also need to check that at the equilibrium, type-A unemployed agents and type-B agents are credit constrained: $u'(c_{u,t+1}^A) > \beta R_{t+1} u'(x_{e,t}^A)$ and $u'(c_{e,t+1}^B) > \beta R_{t+1} u'(c_{e,t}^B)$. Both constraints at the steady state are equivalent to $\beta R < 1$.

We denote by:

$$L_1 = \chi^\varphi(\Omega^A (y^A)^{\varphi+1} + \Omega^B (y^B)^{\varphi+1})$$

the aggregate labor supply for a unitary wage. Using Euler equations and budget constraints, we express consumption solely out of the wage and interest rate. We thus obtain that the Ramsey planner's program can be written as:

$$\begin{aligned}
\max_{(R_t, w_t, B_t)_{t\geq 0}} \omega^A \sum_{t=0}^\infty \beta^t &\left((1+\beta)\log\left(\frac{\chi^\varphi (y^A)^{\varphi+1}}{(1+\varphi)(1+\beta)} w_t^{\varphi+1}\right) + \beta\log(\beta) + \log(R_t) \right) \\
&+ \omega^B \sum_{t=0}^\infty \beta^t \left(\log\left(\frac{\chi^\varphi (w_t y^B)^{\varphi+1}}{1+\varphi}\right) \right), \\
\text{s.t. } G_t + B_{t-1} + (R_t - 1) &\frac{\beta}{(1+\varphi)(1+\beta)} \chi^\varphi \Omega^A (y^A)^{\varphi+1} w_{t-1}^{\varphi+1} + L_1 w_t^{\varphi+1} \\
&= F\left(\frac{\beta}{(1+\varphi)(1+\beta)} \chi^\varphi \Omega^A (y^A)^{\varphi+1} w_{t-1}^{\varphi+1} - B_{t-1}, L_1 w_t^\varphi\right) + B_t,
\end{aligned}$$

with additional constraints $w_t, R_t > 0$. Denoting by $\beta^t \mu_t$ the Lagrangian of the governmental budget constraint, we obtain the following FOCs:

$$\frac{\omega^A}{\Omega^A} = \mu_t \frac{\beta R_t}{(1+\varphi)(1+\beta)}\chi^{\varphi}(y^A)^{\varphi+1}w_{t-1}^{\varphi+1},$$

$$\mu_t = \beta\mu_{t+1}(1+F_{K,t+1}),$$

$$0 = (1+\varphi)(\omega^A(1+\beta)+\omega^B) + \mu_t w_t^{\varphi+1}L_1\left(-1+\varphi(\frac{F_{L,t}}{w_t}-1)\right)$$
$$+\mu_{t+1}\frac{\beta}{1+\beta}\chi^{\varphi}\Omega^A(y^A)^{\varphi+1}w_t^{\varphi+1}\beta(F_{K,t+1}+1-R_{t+1}).$$

At the steady state, we obtain:

$$\frac{\omega^A}{\Omega^A} = \frac{\beta R\mu}{(1+\varphi)(1+\beta)}\chi^{\varphi}(y^A)^{\varphi+1}w^{\varphi+1}, \tag{131}$$

$$1 = \beta(1+F_K) \tag{132}$$

$$0 = (1+\varphi)(\omega^A(1+\beta)+\omega^B) + \mu w^{\varphi+1}L_1\left(-1+\varphi(\frac{F_L}{w}-1)\right) \tag{133}$$
$$+\frac{\beta\mu}{1+\beta}\chi^{\varphi}\Omega^A(y^A)^{\varphi+1}w^{\varphi+1}(1-\beta R).$$

Combing FOCs (131) and (133) implies with $\omega^A+\omega^B=1$:

$$1-\beta R = \omega^B - \omega^A(1+\beta)\frac{\Omega^B(y^B)^{\varphi+1}}{\Omega^A(y^A)^{\varphi+1}} + \omega^A\varphi(1+\beta)\left(1+\frac{\Omega^B(y^B)^{\varphi+1}}{\Omega^A(y^A)^{\varphi+1}}\right)\left(\frac{F_L}{w}-1\right),$$

which is equation (46) using the notation $\Lambda = \frac{\Omega^B(y^B)^{\varphi+1}}{\Omega^A(y^A)^{\varphi+1}}$.

F The General Model

F.1 Deriving FOCs

The Lagrangian of the Ramsey program (50)–(55) can be written as:

$$\begin{aligned}\mathcal{L} = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{f=1}^{F} \omega^f m^f \int_i u(x_{i,t}^f)\ell(di) \\ & - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{f=1}^{F} m^f \int_i \left(\lambda_{i,t}^f - (1+r_t)\lambda_{i,t-1}^f\right) u'(x_{i,t}^f)\ell(di) \\ & - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \Bigg(G_t + T_t + r_t A_{t-1} + \left(\frac{1}{\tilde{\tau}_t} + \frac{1}{1/\varphi+1}\right) \frac{l_t^{1/\varphi+1}}{\chi} \sum_{f=1}^{F} m^f \int_i (y_{i,t}^f)^{\tilde{\tau}_t} \ell(di) \\ & \quad - F\left(\sum_{f=1}^{F} m^f \int_i a_{i,t-1}^f \ell(di) - B_{t-1}, l_t \sum_{f=1}^{F} m^f \int_i (y_{i,t}^f)^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}} \ell(di)\right) - B_t + B_{t-1}\Bigg),\end{aligned} \tag{134}$$

with the additional positivity constraints $x_{i,t}^f, l_{i,t}^f \geq 0$ that always hold.

FOC with respect to public debt B_t. Deriving (134) with respect to B_t yields: $\mu_t = \beta(1+\tilde{r}_{t+1})\mu_{t+1}$, which is FOC (59).

FOC with respect to savings choices $a_{i,t}^f$. Using $\frac{\partial x_{j,t}^g}{\partial a_{i,t}^f} = -1_{i=j,f=g}$ and $\frac{\partial x_{j,t+1}^g}{\partial a_{i,t}^f} = (1+r_{t+1})1_{i=j,f=g}$, and the notation $\psi_{i,t}$ of (56), deriving (134) with respect to $a_{i,t}$ yields for unconstrained agents: $\psi_{i,t} = \beta\mathbb{E}_t\left[R_{t+1}\psi_{i,t+1}\right] + \beta\mathbb{E}_t\left[\mu_{t+1}(1+\tilde{r}_{t+1} - R_{t+1})\right]$, while for constrained agents ($a_{i,t}=0$), we have $\lambda_{i,t}=0$. Using FOC (59) and the notation $\hat{\psi}_{i,t}$ of (57), we deduce for unconstrained agents: $\hat{\psi}_{i,t}^f = \beta\mathbb{E}_t[R_{t+1}\hat{\psi}_{i,t+1}^f]$, which is FOC (58).

FOC with respect to the interest rate R_t. Since $\frac{\partial x_{i,t}^f}{\partial r_t} = a_{i,t-1}^f$, deriving (134) with respect to R_t yields: $0 = \sum_{f=1}^{F} m^f \int_i \left(\hat{\psi}_{i,t} a_{i,t-1}^f + \lambda_{i,t-1}^f u'(x_{i,t}^f)\right)\ell(di)$, which is FOC (60).

FOC with respect to labor supply l_t. Using $\frac{\partial x_{i,t}^f}{\partial l_t} = \frac{1}{\chi\tilde{\tau}_t}\left(1/\varphi+1\right) l_t^{1/\varphi}(y_{i,t}^f)^{\tilde{\tau}_t}$, deriving (134) with respect to l_t yields:

$$(1/\varphi+1)\frac{l_t^{1/\varphi}}{\chi\tilde{\tau}_t}\sum_{f=1}^{F} m^f \int_i (y_{i,t}^f)^{\tilde{\tau}_t}\hat{\psi}_{i,t}\ell(di) = \mu_t \sum_{f=1}^{F} m_f \sum_{j=1}^{J} s_j^f (y_j^f)^{\tilde{\tau}_t}\left(\frac{l_t^{1/\varphi}}{\chi} - (y_j^f)^{\frac{\tilde{\tau}_t}{1/\varphi+1}} F_L(K_t, L_t)\right),$$

which is FOC (61).

FOC with respect to progressivity $\tilde{\tau}_t$. Using $\frac{\partial x_{i,t}^f}{\partial l_t} = \frac{1}{\chi \tilde{\tau}_t} (1/\varphi + 1) l_t^{1/\varphi} (y_{i,t}^f)^{\tilde{\tau}_t} (-\tilde{\tau}_t^{-1} + \log y_{i,t}^f)$, the derivative of (134) with respect to $\tilde{\tau}_t$ is:

$$0 = \frac{l_t^{1/\varphi+1}}{\chi \tilde{\tau}_t} \sum_{f=1}^F m^f \int_i \hat{\psi}_{i,t} (y_{i,t}^f)^{\tilde{\tau}_t} \left(\log y_{i,t}^f - \frac{1}{\tilde{\tau}_t} \right) \ell(di)$$
$$- \frac{\mu_t l_t}{1/\varphi + 1} \sum_{f=1}^F m^f \sum_{j=1}^J s_j^f \log y_j^f \left(\frac{l_t^{1/\varphi}}{\chi} (y_j^f)^{\tilde{\tau}_t} - (y_j^f)^{\frac{1/\varphi+1+\tilde{\tau}_t}{(1/\varphi+1)}} F_{L,t} \right),$$

which is FOC (63).

F.2 Consistency of the Two Approaches

We verify here that the analytical approach of Section 3.1 and the quantitative approach of Section 4) yield consistent results in the case of GHH log utility. We proceed in two steps. First, in Section F.2.1, we check that the application of the Lagrangian approach to the environment of Section 3.1 delivers the same FOCs as in the analytical approach (equations (88)–(90)). Second, in Section F.2.2, we compare the quantitative outcomes of the two approaches and show that the analytical solution is the limit of the general solution when the transition matrix converges to the anti-diagonal matrix.

F.2.1 Checking that FOCs are Identical

We check here that the FOCs of the Ramsey program derived in the general case of Section 4.1 (i.e., equations (58)–(63)) exactly simplify to the FOCs derived in the specific case of Section 3.1 (i.e., equations (88)–(90)). The larger number of equations in the first case comes from the definitions of Lagrange multipliers and additional instruments. We start with expressing $\psi_{i,t}$ in the context of the log utility function ($u = \log$). The expression (56) of $\psi_{i,t}$ becomes:

$$\psi_{i,t} x_{i,t} = 1 + (\lambda_{i,t} - R_t \lambda_{i,t-1}) \frac{1}{x_{i,t}}. \tag{135}$$

We now turn to the FOCs. Note that FOC (59) is exactly the same as FOC (88), while FOC (63) has no equivalent in the simplified version since the progressivity parameter τ_t is set to one (or $\tilde{\tau}_t$ to $1 + \varphi$). The three remaining FOCs are equations (58), (60), and (61) for which we only have one type ($F = 1$) and two types. Taking advantage of the deterministic transitions, as well as the fact that unemployed agents are credit-constrained with null productivity, these FOCs can also be written as:

$$\psi_{e,t} - \mu_t = \beta R_{t+1}(\psi_{u,t+1} - \mu_{t+1}), \tag{136}$$

$$\mu_t x_{u,t} = \psi_{u,t} x_{u,t} + \frac{\lambda_{e,t-1}}{a_{e,t-1}}, \tag{137}$$

$$\mu_t x_{e,t} = \psi_{e,t} x_{e,t} + \mu_t x_{e,t} \varphi \left(1 - \frac{F_{L,t}}{w_t}\right), \tag{138}$$

while similarly expressions of $\psi_{i,t}$ in (135) can further be specified as:

$$\psi_{e,t} x_{e,t} = 1 + \frac{\lambda_{e,t}}{x_{e,t}}, \tag{139}$$

$$\psi_{u,t} x_{u,t} = 1 - R_t \lambda_{e,t-1} \frac{1}{x_{u,t}}. \tag{140}$$

Combining (137) and (140) with $a_{e,t-1} = \frac{x_{u,t}}{R_t}$ (unemployed budget constraint) gives:

$$\mu_t x_{u,t} = 1, \tag{141}$$

which is, with the expression of $x_{u,t} = R_t \frac{\beta}{1+\beta} \frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi}$, identical to FOC (89).

Using the consumption Euler equation (20) stating that $x_{u,t+1} = \beta R_{t+1} x_{e,t}$, the budget constraints (18) and (19), and equation (141) meaning that $1 = \beta \mu_{t+1} R_{t+1} x_{e,t}$, we deduce from (136) and (139):

$$\frac{\lambda_{e,t}}{x_{e,t}} = \frac{\beta}{1+\beta}(\mu_t x_{e,t} - 1). \tag{142}$$

Finally, we turn to FOC (138). Combined with the expressions of $\psi_{e,t}$ in (139), and of $\lambda_{e,t}$ in (142), this becomes:

$$x_{e,t}\mu_t \left(1 - (1+\beta)\varphi \frac{\tau_t^L}{1 - \tau_t^L}\right) = 1. \tag{143}$$

Using the budget constraint (18) stating that $x_{e,t} = \frac{w_t(\chi w_t)^{\varphi}}{(1+\beta)(1+\varphi)}$, equation (143) becomes FOC (90) (or more precisely its equivalent representation). This completes the proof that the generic FOCs of Section 4.1 exactly imply the FOCs (88)–(90).

F.2.2 Comparing the Quantitative Outcomes of the two Approaches

We show that the analytical solution can be computed as the limit of the quantitative model where the transition matrix converges to the anti-diagonal matrix of Assumption A. We thus consider a specification of the quantitative model that is similar to the one of the analytical model: a GHH utility function, a linear labor tax, a two-state productivity process, and a zero credit constraint. We consider the transition matrix Π_ε defined for any $\varepsilon \in [0,1]$ as: $\Pi_\varepsilon = \begin{bmatrix} \varepsilon & 1-\varepsilon \\ 1-\varepsilon & \varepsilon \end{bmatrix}$, which for $\varepsilon = 0$ corresponds to the anti-diagonal case of Assumption A.

We use the same calibration as in Figure 4, namely: $\alpha = 0.3$, $\beta = 0.7$, $\varphi = 0.3$, $\delta = 1$, $G = 0.01$, $\chi = 1$. This calibration guarantees the existence of a positive debt and a positive capital tax in the analytical model (when $\varepsilon = 0$). We compute the optimal steady-state fiscal policy as a function of ε with the truncation approach, as in Section 4. We plot the results in Figure 5. The first observation is for low values of ε (from 10^{-6} to 10^{-10}): the outcomes of the two models are very similar. The quantitative resolution is thus consistent with the analytical method. The second observation is when ε increases beyond 10^{-5}, the capital tax diminishes sharply, while the labor tax goes up. This result is consistent with intuition. Indeed, in this very stylized setup, a higher ε means that a higher share of the population remains unemployed with a null income. Their sole resource is their savings. Diminishing the capital tax fosters savings and enables agents to better self-insure themselves against the null income risk. Increasing the labor tax enables the government to balance its budget – since public spending remains fixed.

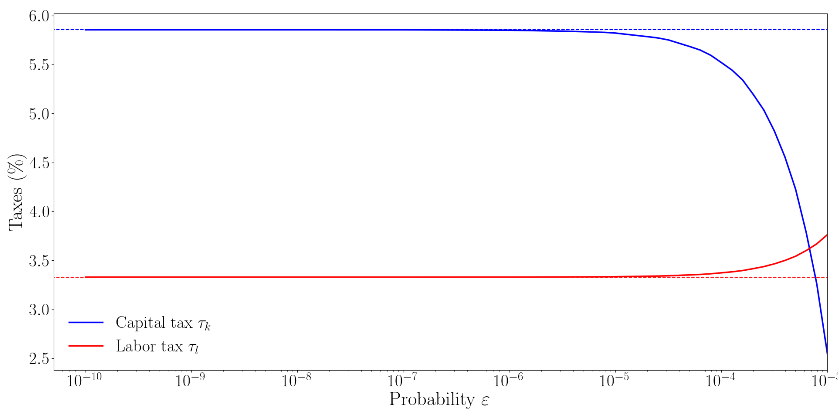

Figure 5: Comparison of the results of the quantitative model (plain lines) to those of the analytical model (dashed lines).

G The Ramsey Program on the Truncated Model

The (refined) *truncation* consists in expressing the model in terms of groups of agents (called *truncated histories*, because they share the same recent idiosyncratic histories) instead of individual agents. The resulting model, called the *truncated model* has the major advantage of admitting a finite state-space representation, which lends itself to the computation of optimal Ramsey policies. In Section G.1, we explain how to construct truncated histories. In Section G.2, we construct the truncated model via a proper aggregation of the underlying model. We also express the Ramsey program in this model and compute the associated FOCs in this model. Finally, in Section G.3, we explain how we use the inverse optimal approach in the truncated Ramsey model to compute the weights of the SWF based on a given fiscal system.

G.1 The Refined Truncation: Definition and Construction

G.1.1 Definitions

Agents face an idiosyncratic risk y, whose realizations belong to a finite set $\mathcal{Y}$ of cardinal $|\mathcal{Y}|$. A possible history in period t is a sequence of idiosyncratic states: $y^t = (y_0, \ldots, y_t) \in \mathcal{Y}^{t+1}$. The probability to transit from an history $y^t = (y_0, \ldots, y_t)$ at date t to an history $\tilde{y}^{t+1} = (\tilde{y}_0, \ldots, \tilde{y}_{t+1})$ at date $t+1$ is equal to $\mathbf{1}_{(y_0,\ldots,y_t)=(\tilde{y}_0,\ldots,\tilde{y}_t)}\Pi^{\mathcal{Y}}_{y_t\tilde{y}_{t+1}}$, where $\Pi^{\mathcal{Y}}_{y_t\tilde{y}_{t+1}}$ is the probability to switch from productivity level y_t to productivity level $\tilde{y}_{t+1}$ and $\mathbf{1}_{(y_0,\ldots,y_t)=(\tilde{y}_0,\ldots,\tilde{y}_t)}$ is equal to 1 if $\tilde{y}^{t+1}$ is a continuation of y^t and 0 otherwise. In the case of ex-ante heterogeneity, the matrix $\Pi^{\mathcal{Y}}$ can be constructed from the matrices Π^f (see Section 2).

We consider a set $\mathcal{H}$ of vectors of elements of $\mathcal{Y}$ that are possibly of different lengths. Each element of $\mathcal{H}$, denoted by h and of length N_h, is called a *truncated history* and represents a possible history over the last N_h periods.

The set $\mathcal{H}$ will be called a *partition* of the set of idiosyncratic histories if at any date t sufficiently large (including at the steady state $t = \infty$), each agent can be assigned to one and only one truncated history. More precisely, for any t sufficiently large, for any history $(y_0, \ldots, y_t) \in \mathcal{Y}^{t+1}$, there must be a unique $h \in \mathcal{H}$ such that $(y_{t-N_h+1}, \ldots, y_t) = h$. In other words, an history h is a vector of productivity levels, where the last element represents the current productivity level and previous elements represent productivity levels in earlier periods.

We can then compute the transition probabilities between any two elements h and $\tilde{h}$ (in the next period) of the partition $\mathcal{H}$ and we denote this probability $\Pi_{h\tilde{h}}$. If $h = (y_{h,-N_h}, \ldots, y_{h,0})$ and $\tilde{h} = (y_{\tilde{h},-N_{\tilde{h}}}, \ldots, y_{\tilde{h},0})$, the probability $\Pi_{h\tilde{h}}$ can be defined as $\Pi_{h\tilde{h}} = \mathbf{1}_{(y_{h,\min(-N_h,-N_{\tilde{h}})+1},\ldots,y_{h,0})=(y_{\tilde{h},\min(-N_h,-N_{\tilde{h}})},\ldots,y_{\tilde{h},1})}\Pi_{y_{h,0}y_{\tilde{h},0}}$, i.e. the probability to transit from productivity level $y_{h,0}$ to $y_{\tilde{h},0}$ and $\tilde{h}$ being a possible continuation of h.

Finally, a *truncation* is a partition where $(\Pi_{h\tilde{h}})_{h,\tilde{h}\in\mathcal{H}}$ is a proper transition matrix, i.e., such that $\Pi_{h\tilde{h}} \geq 0$ (which holds by construction of the probabilities) and for all h, $\sum_{\tilde{h}\in\mathcal{H}} \Pi_{h\tilde{h}} = 1$. The latter condition does not hold for arbitrary partitions. For instance, if $\mathcal{Y} = \{y_1, y_2\}$, then $\{(y_1, y_1), (y_1, y_2, y_1), (y_2, y_2, y_1), (y_2)\}$ is a partition of idiosyncratic histories but does not imply a well-defined transition matrix (the probabilities of the transitions out of (y_2) do not sum to 1). Using the transition probability, we can compute the size of truncated histories as the stationary distribution associated to the transition matrix Π. More precisely, the vector of truncated history sizes (S_h) is defined as:[36]

$$S_h = \sum_{\tilde{h}\in\mathcal{H}} S_{\tilde{h}} \Pi_{\tilde{h}h}. \tag{144}$$

LeGrand and Ragot (2022a) consider a *uniform truncation* $\mathcal{H}$, where all truncated histories have the same length N. this case, $\mathcal{H}$ is identical to $\mathcal{Y}^N$ and its cardinal grows exponentially with N. LeGrand and Ragot (2022b) consider a *refined truncation,* where truncation lengths can vary from one truncated history to another. However, their construction is limited to two idiosyncratic states and we propose here a generalization of the construction of a refined truncation to an arbitrary number of productivity states.

G.1.2 Construction of a refined truncation

The construction of the refined truncation is based on the observation that the vast majority of idiosyncratic processes considered in the literature are persistent. This means that their discretization in a finite number of productivity levels implies a transition matrix with a dominant diagonal: $\Pi^{\mathcal{Y}}_{yy} > \Pi^{\mathcal{Y}}_{y\tilde{y}}$ for all $\tilde{y} \in \mathcal{Y}, \tilde{y} \neq y$. A consequence of this persistence is that in a uniform truncation, truncated histories with constant productivity (i.e., of the type $(y, \ldots, y)$ with the same y at all dates) are of much larger sizes (as defined in equation (144)). The refined truncation consists then in splitting these large histories into smaller ones, so as to obtain a partition with a reduced disparity in sizes.

The refined truncation can be constructed recursively starting from a uniform truncation of length N. The initial truncation is then the set of truncated histories $\{(y_1, \ldots, y_1), \ldots, (y_i, \ldots, y_i), \ldots, (y_{|\mathcal{Y}|}, \ldots, y_{|\mathcal{Y}|})\}$ with $|\mathcal{Y}|^N$ elements. The first step of the refinement consists in splitting each of the constant truncated history of length N into $|\mathcal{Y}|$ truncated histories of length $N + 1$. For any i, the N-vector $(y_i, \ldots, y_i)$ is split into $|\mathcal{Y}|$ vectors of length

[36] In the case of ex-ante heterogeneity, the matrix Π^h is block-diagonal, and hence the associated Markov process is not irreducible. In that case, the vector of history sizes may not be unique (S_h). One possibility is to carefully normalize (S_h) by ex-ante type (such that for any type f, the associated truncated histories sum to the size m^f). Another option is apply the procedure of Section G.1.2 to each type (i.e., each block of the transition matrix) and concatenate the resulting truncations of the different blocks.

$N+1$: $(y, y_i, \ldots, y_i)$ for all $y \in \mathcal{Y}$ – among which there is one constant $N+1$-vector $(y_i, \ldots, y_i)$. After the first refinement step, the refinement for each productivity level adds $|\mathcal{Y}|-1$ elements ($|\mathcal{Y}|$ additions and one deletion), such that the cardinal of the partition is $|\mathcal{Y}|^N - |\mathcal{Y}| + |\mathcal{Y}|^2$. The second step consists in splitting each of the $|\mathcal{Y}|$ constant $N+1$-vectors $(y_i, \ldots, y_i)$ into $|\mathcal{Y}|$ vectors of length $N+2$, as in the first step. The refinement is possible at any step since the previous step always introduces a constant productivity vector. The refinement stops when the desired refined truncation length is achieved for each constant productivity vector.

The refined truncation length can differ from one productivity level to another and the refined truncation is characterized by a uniform truncation length N and a vector of a refined truncation lengths $(N_1, \ldots, N_{|\mathcal{Y}|})$ – where $N_i \geq N$ is the truncation length of the vector of constant productivity y_i and is assumed to be greater than the uniform truncation length. The resulting truncation counts $|\mathcal{Y}|^N + (|\mathcal{Y}|-1)\sum_{i=1}^{|\mathcal{Y}|}(N_i - N)$ elements, which therefore grows linearly in the lengths $(N_1, \ldots, N_{|\mathcal{Y}|})$. The refinement is thus a parsimonious method to reduce the size of the largest elements of the partition, while keeping a reasonable number of partition elements.

Example of the construction of a refined truncation. To illustrate the truncation, consider a productivity set $\mathcal{Y} = \{y_1, y_2, y_3\}$ with $|\mathcal{Y}| = 3$ elements and a refined truncation with length parameters $N = 2$ and $(N_1, N_2, N_3) = (2, 3, 4)$. We detail the recursive construction of the refined partition.

0. We start with the uniform partition with $3^2 = 9$ elements, that is denoted $\mathcal{H}_0 = \{\boldsymbol{(y_1, y_1)}, (y_1, y_2), (y_1, y_3), (y_2, y_1), \boldsymbol{(y_2, y_2)}, (y_2, y_3), (y_3, y_1), (y_3, y_2), \boldsymbol{(y_3, y_3)}\}$. We recall that the truncated history (y_2, y_3) gathers all agents, who have the productivity level y_3 in the current period and y_2 in the previous one.

1. In the first step, we split the constant productivity vectors (y_i, y_i) $(i = 2, 3)$ by adding one past productivity level. The truncated history (y_1, y_1) does not need to be further refined since $N_1 = N = 2$. The truncated history (y_i, y_i) for $i = 2, 3$ is refined into $\{(y_1, y_i, y_i), (y_2, y_i, y_i), (y_3, y_i, y_i)\}$. The updated truncation is:

$$\begin{aligned}\mathcal{H}_1 = \{&\boldsymbol{(y_1, y_1)}, (y_1, y_2), (y_1, y_3), (y_2, y_1), \\ &(y_1, y_2, y_2), \boldsymbol{(y_2, y_2, y_2)}, (y_3, y_2, y_2), \\ &(y_2, y_3), (y_3, y_1), (y_3, y_2), \\ &(y_1, y_3, y_3), (y_2, y_3, y_3), \boldsymbol{(y_3, y_3, y_3)}\}.\end{aligned}$$

2. Since $N_2 = 3$, the truncated history (y_2, y_2, y_2) does not need to be further split. In the second step, only (y_3, y_3, y_3) is refined into $\{(y_1, y_3, y_3, y_3), (y_2, y_3, y_3, y_3), (y_3, y_3, y_3, y_3)\}$. This is the final step as (y_3, y_3, y_3, y_3) has $N_3 = 4$ elements. The final truncation is:

$$\begin{aligned}\mathcal{H} = \{&\boldsymbol{(y_1, y_1)}, (y_1, y_2), (y_1, y_3), (y_2, y_1),\\ &(y_1, y_2, y_2), \boldsymbol{(y_2, y_2, y_2)}, (y_3, y_2, y_2),\\ &(y_2, y_3), (y_3, y_1), (y_3, y_2),\\ &(y_1, y_3, y_3), (y_2, y_3, y_3),\\ &(y_1, y_3, y_3, y_3), (y_2, y_3, y_3, y_3), \boldsymbol{(y_3, y_3, y_3, y_3)}\},\end{aligned}$$

which has 15 elements, consistently with the formula $|\mathcal{Y}|^N + (|\mathcal{Y}| - 1) \sum_{i=1}^{|\mathcal{Y}|}(N_i - N)$.

The implementation of this algorithm is done in Julia in a functional way that allows the code to stay close from the recursive algorithm we have just described.

G.2 The Ramsey program in the truncated model

We now provide the solution for optimal policy on the truncated model.

Aggregating the Bewley model. Constructing the truncated model requires to solve the Bewley model at the steady state and then aggregate the solution in terms of truncated histories instead of agents.

The first step is the model resolution at steady state for a given fiscal policy. Solving the model of Section 2 (using standard methods such as EGM) yields the steady-state wealth distribution as well as policy functions. The wealth distribution is denoted $\Lambda : \mathbb{R}_+ \times \mathcal{Y} \to \mathbb{R}_+$, such that $\Lambda(da, y)$ is the distribution of agents with wealth in $[a, a + da)$ and productivity y. The policy rules for savings, consumption, labor, and the Lagrange multiplier on the credit constraint (ν in equation 10) are denoted by g_a, g_c, g_l and g_ν and are mappings from $\mathbb{R}_+ \times \mathcal{Y} \to \mathbb{R}_+$. For instance, $g_c(a, y)$ is the current consumption level of an agent endowed with the beginning-of-period wealth a and productivity y.[37]

For the second step, which consists in constructing the truncated model, we consider a set of truncated histories $\mathcal{H}$, as well as the associated transition matrix $(\Pi_{h\tilde{h}})_{h,\tilde{h}\in\mathcal{H}}$ and the corresponding vectors of length sizes (S_h). The construction of the truncated model aims at attributing to each history $h \in \mathcal{H}$ an allocation that verifies budget constraints and

[37] Actually, because of the GHH assumption, the policy function for labor could simply be seen as a function of the current productivity level.

FOCs at the truncated-history level. Consider an history $h = (y_{h,N_h-1}, \ldots, y_{h,0})$. The first step is to compute the distribution over asset choices and histories (and not productivity level). The construction is recursive. We start from the distribution $\Lambda(\cdot, y_{h,N_h-1})$ and apply the policy rule $g_a(\cdot, y_{h,N_h-2})$ to obtain the steady-state wealth distribution of agents with history $(y_{h,N_h-1}, y_{h,N_h-2})$ that we still denote $\Lambda(\cdot, (y_{h,N_h-1}, y_{h,N_h-2}))$. We then apply the policy rules corresponding to the following productivity levels of h and derive the steady-state wealth distribution of agents with history h – denoted as $\Lambda(\cdot, h)$.

The next step is to use this distribution to aggregate the steady-state model. First, the mass of agents experiencing each history $h \in \mathcal{H}$ is $S_h = \int_0^{+\infty} \Lambda(da, h)$, which is identical to the computation in equation (144). Second, we compute the per-capita allocation for each truncated history h. The per-capita consumption c_h, beginning-of-period saving $\tilde{a}_h$, end-of-period saving a_h, and Lagrange multiplier value can be defined as follows:

$$z_h := \int_0^{\infty} g_x(a, y_{h,0})\Lambda(da, h)/S_h, \text{ for } z = c, a, l, \tag{145}$$

$$\tilde{a}_h := \int_0^{\infty} a\Lambda(da, h)/S_h, \tag{146}$$

$$\nu_h := \int_{-\bar{a}}^{+\infty} g_\nu(a, y_1)\, \Lambda(da, h)/S_h. \tag{147}$$

With the GHH assumption, the history-specific labor supply actually expresses as: $l_h = (\chi(1-\tau)\, w)^{\frac{1}{1/\varphi+\tau}} (y_h)^{\frac{1-\tau}{1/\varphi+\tau}}$. We define the set of credit constrained histories $\mathcal{C}_{\mathcal{H}}$ as the non-empty set of histories in h such that: (i) the measure of credit-constrained histories is as close as possible to the measure of credit-constrained agent in the underlying model and (ii) the credit-constrained histories have the largest value of ν_h.

From the individual budget constraint (8), we construct history-specific budget constraints:

$$c_h + a_h = w(l_h y_h)^{1-\tau_t} + (1+r)\tilde{a}_h. \tag{148}$$

We also define an history-specific aggregation parameter ξ_h^u:

$$\xi_h^u := \frac{\int_0^{\infty} u\left(g_c(a, y_1) - \frac{\chi^{-1}}{1+1/\varphi} g_l(a, y_1)^{1+1/\varphi}\right) \Lambda(da, h)}{u(c_h - \chi^{-1}\frac{l_h^{1+1/\varphi}}{1+1/\varphi})}, \tag{149}$$

such that the aggregate period utility of agents having a history h is the period utility of the aggregate consumption and labor multiplied by ξ_h^u. This parameter captures the interaction between the non-linearity of the function u and the heterogeneity within h.

We similarly define history-specific Euler-equation parameters ξ_h^E:

$$\xi_h^E u'(c_h - \chi^{-1}\frac{l_h^{1+1/\varphi}}{1+1/\varphi}) = \beta(1+r)\left[\sum_{h'\in\mathcal{H}} \Pi_{h,h'}\xi_{h'}^E u'(c_{h'} - \chi^{-1}\frac{l_{h'}^{1+1/\varphi}}{1+1/\varphi})\right] + \nu_h, \tag{150}$$

that guarantee that Euler equations hold for truncated histories.

The allocation $(c_h, l_h, a_h, \tilde{a}_h, \nu_h)_h$ given by equations (145)–(147), the budget constraint (148), the Euler equation (150) characterize together with the set of credit-constrained histories $\mathcal{C}_{\mathcal{H}}$ and the parameters $(\xi_h^u)_h$ the truncated model for a given fiscal policy. Every history h in the truncated model acts a "representative agent" with their own budget constraint and their own Euler equation. The history-wise allocation is a solution of the truncated model (with the within-heterogeneity parameters ξ_h^u and ξ_h^E).

By construction, the prices and aggregate quantities (capital, labor, and consumption) are the same in the truncated model as in the individual Bewley model. For the ability of this aggregated model to capture the dynamics with aggregate shocks, see LeGrand and Ragot (2022a) and LeGrand and Ragot (2022b), and Appendix H for the current model.

Ramsey problem. We now use the truncation to solve the Ramsey program. See LeGrand and Ragot (2023) for the ability of the method to compute optimal policies.

We express the Ramsey program with given weights (ω^f) and given within-heterogeneity parameters $(\xi_h^u, \xi_h^E)_h$, and then derive the FOCs. We then guess a fiscal policy for which we compute the truncated model (and the parameters $(\xi_h^u, \xi_h^E)_h$). We verify that whether the Ramsey FOCs hold for this fiscal policy, in which case it is optimal – or update the fiscal policy (and the truncated model).

The objective of the planner is: $W_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{f=1}^{F} \omega^f \sum_{h^f\in\mathcal{H}^f} S_{h^f}\xi_{h^f}^u u(c_{t,h^f} - \chi^{-1}\frac{l_{h^f}^{1+1/\varphi}}{1+1/\varphi})$, where we have separated the histories of the F types of agents, to explicit the role of social weights. To simplify the exposition we will write $\omega^{f(h)}$, or simply ω^f when no confusion is possible, for the social weight of an agent of type f having an history $h \in \mathcal{H}$. The objective of the planner can thus be written as $W_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{h\in\mathcal{H}} \omega_{f(h)} S_h \xi_h^u u\left(c_{t,h} - \chi^{-1}\frac{l_h^{1+1/\varphi}}{1+1/\varphi}\right)$,where $\omega_{f(h)}$ are actually type-dependent and *not* history-dependent. The Ramsey problem can be expressed as follows:

$$\max_{\left(r_t,w_t,\tau_t,l_t,B_t,K_t,L_t,(a_{t,h},,c_{t,h},l_{t,h},\nu_{t,h})_{h\in\mathcal{H}}\right)_{t\geq 0}} W_0 \tag{151}$$

$$\text{s.t. } G_t + (1+r_t)B_{t-1} + r_t K_{t-1} + w_t \sum_h (l_{t,h}y_h)^{1-\tau_t} = F(K_{t-1}, L_t, z_t) + B_t, \tag{152}$$

and subject to:

$$\text{for all } h \in \mathcal{H}\text{: } c_{t,h} + a_{t,h} = w_t (l_{t,h} y_h)^{1-\tau_t} + (1+r_t)\tilde{a}_{t,h} + T_t, \tag{153}$$

$$\xi_h^E u'(c_{t,h} - \frac{\chi^{-1} l_{t,h}^{1+1/\varphi}}{1+1/\varphi}) = \beta \mathbb{E}_t \left[(1+r_{t+1}) \sum_{h' \in \mathcal{H}} \Pi_{hh'} \xi_{h'}^E u'(c_{t+1,h'} - \frac{\chi^{-1} l_{t+1,h'}^{1+1/\varphi}}{1+1/\varphi})\right] + \nu_{t,h}, \tag{154}$$

$$a_{t,h} \geq 0,\ \nu_{t,h}(a_{t,h} + \bar{a}) = 0,\ \nu_{t,h} \geq 0,\ c_{t,h} \geq 0, \tag{155}$$

$$l_{t,h} = (\chi (1-\tau_t) w_t)^{\frac{1}{1/\varphi + \tau_t}} (y_h)^{\frac{1-\tau_t}{1/\varphi+\tau_t}}, \tag{156}$$

$$\tilde{a}_{t,h} = \sum_{\tilde{h} \in \mathcal{H}} \Pi_{\tilde{h}h,t} \frac{S_{,\tilde{h}}}{S_h} a_{t-1,\tilde{h}}, \tag{157}$$

$$K_t + B_t = \sum_h S_{t,h} a_{t,h},\ L_t = \sum_h S_{t,h} y_h l_{t,h}. \tag{158}$$

As in the initial case, one can simplify the program with the change of variables: $x_{t,h} := c_{t,h} - \chi^{-1} \frac{(l_{t,h})^{1+1/\varphi}}{1+1/\varphi}$, $l_t := (\chi (1-\tau_t) w_t)^{\frac{1}{1/\varphi+\tau_t}}$ and $\tilde{\tau}_t := \frac{(1/\varphi+1)(1-\tau_t)}{1/\varphi+\tau_t}$. We now factorize the Ramsey program (151)–(158) as in Section F by introducing Lagrange multipliers $\lambda_{t,h}$ on history-specific Euler equations (154). The new Ramsey objective is:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{h \in \mathcal{H}} \left[\omega^f S_h \xi_h^u u(x_{t,h}) - \lambda_{c,t,h} \xi_h^{u,E} u'(x_{t,h}) + \tilde{\lambda}_{t,h}(1+r_t) \xi_h^{u,E} u'(x_{t,h})\right],$$

with $\tilde{\lambda}_{t,h} = \frac{1}{S_{t,h}} \sum_{\tilde{h} \in \mathcal{H}} S_{t-1,\tilde{h}} \lambda_{t-1,\tilde{h}} \Pi_{t,\tilde{h},h}$. The new Ramsey program (151)–(158) is:

$$\max_{\left(r_t, w_t, \tilde{\tau}_t, l_t, B_t, K_t, L_t, (a_{t,h,}, x_{t,h}, \nu_{t,h})_{h \in \mathcal{H}}\right)_{t \geq 0}} J, \tag{159}$$

$$\text{s.t.} G_t + r_t A_{t-1} + \left(\frac{1}{\tilde{\tau}_t} + \frac{1}{1/\varphi+1}\right) \frac{l_t^{1/\varphi+1}}{\chi} \sum_{y \in \mathcal{Y}} S_y y^{\tilde{\tau}_t} = F(A_{t-1} - B_{t-1}, L_t) + B_t - B_{t-1}, \tag{160}$$

$$\text{for all } h \in \mathcal{H}\text{: } x_{t,h} + a_{t,h} = (1+r_t)\tilde{a}_{t-1} + \frac{1}{\chi \tilde{\tau}_t} l_t^{1+1/\varphi} (y_{t,h})^{\tilde{\tau}_t}, \tag{161}$$

$$a_t \geq -\bar{a},\ \nu_{t,h}(a_{i,t,h}^f + \bar{a}) = 0,\ \nu_{t,h} = \beta \mathbb{E}_t \left[(1+r_{t+1}) u'(x_{h',t+1})\right] - u'(x_{t,h}), \tag{162}$$

$$\tilde{a}_{t,h} = \sum_{\tilde{h} \in \mathcal{H}} \Pi_{\tilde{h}h,t} \frac{S_{,\tilde{h}}}{S_h} a_{t-1,\tilde{h}},\ \tilde{\lambda}_{t,h} = \frac{1}{S_{t,h}} \sum_{\tilde{h} \in \mathcal{H}} S_{t-1,\tilde{h}} \lambda_{t-1,\tilde{h}} \Pi_{t,\tilde{h},h}, \tag{163}$$

$$A_t = \sum_{h \in \mathcal{H}} S_h a_{h,t},\ L_t = l_t \sum_{y \in \mathcal{Y}} S_y y^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}}, \tag{164}$$

FOCs of the Planner. We denote by $\beta^t \mu_t$ the discounted Lagrange multiplier on the government budget constraint (160). We use the history-specific budget constraint (161) to substitute for $x_{t,h}$. The FOCs are then computed as derivatives with respect to $a_{t,h}$ (for unconstrained agent), B_t, l_t, r_t, and $\tilde{\tau}_t$. Similarly to the individual case in equations (56) and (57), we define $\hat{\psi}_{h,t} := \mu_t - \left(\omega^f \xi_h^u u'(x_{h,t}) - \left(\lambda_{h,t} - (1+r_t)\tilde{\lambda}_{h,t}\right) u''(x_{h,t})\right)$. Denoting by $\mathcal{C}_{t,\mathcal{H}}$ as the set of credit constrained history at date t, the FOCs of (159)–(164) are:

$$\begin{aligned}
B_t\colon\quad & \mu_t = \beta \mathbb{E}_t \left[(1+F_{K,t+1})\mu_{t+1}\right], \\
a_{h,t}\colon & \hat{\psi}_{h,t} = \beta \mathbb{E}_t \hat{\psi}_{h',t+1}(1+r_{t+1}) \text{ for } h \notin \mathcal{C}_{t,\mathcal{H}}, \\
l_t\colon\quad & 0 = \tfrac{1+1/\varphi}{\chi \tilde{\tau}_t} l_t^{1/\varphi} \textstyle\sum_{h\in\mathcal{H}} S_h \hat{\psi}^f_{i,h,t} (y_h)^{\tilde{\tau}_t} - \mu_t \textstyle\sum_{h\in\mathcal{H}} S_h \Big(\tfrac{l_t^{1/\varphi}}{\chi} (y_h)^{\tilde{\tau}_t} - (y_h)^{1+\frac{\tilde{\tau}_t}{1/\varphi+1}} F_{L,t} \Big), \\
r_t\colon\quad & 0 = \textstyle\sum_{h\in\mathcal{H}} S_h \left(\hat{\psi}_{i,h,t} \tilde{a}_{h,t-1} - \tilde{\lambda}_{h,t} u'(x_{h,t}) \right), \\
\tilde{\tau}_t\colon\quad & 0 = \tfrac{l_t^{1+1/\varphi}}{\chi \tilde{\tau}_t} \textstyle\sum_{k=1}^{N^{tot}} S_h \hat{\psi}_{h,t} (y_h)^{\tilde{\tau}_t} \left(-\tfrac{1}{\tilde{\tau}_t} + \log y_h \right) \\
& \quad - \mu_t \tfrac{l_t}{1/\varphi+1} \left(\textstyle\sum_{h\in\mathcal{H}} S_h \log y_h \left(\tfrac{l_t^{1/\varphi}}{\chi} (y_h)^{\tilde{\tau}_t} - (y_h)^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}} F_{L,t} \right) \right).
\end{aligned} \tag{165}$$

G.3 Estimation of the social weights

We now provide formulas to derive the parameters ξ^E (equation (169)) and the social weights ω^f (equation (180)). Using simple linear algebra, we derive closed-form expressions.

We assume as given an indexing of histories over $\mathcal{H}$ of cardinal N_{tot} (total number of histories). We denote with a bold letter the N_{tot}-vector associated to a given variable: e.g., $\boldsymbol{S} = (S_h)_{h\in\mathcal{H}}$ is vector of history sizes. Similarly, $\boldsymbol{a}$, $\boldsymbol{c}$, $\boldsymbol{l}$, and $\boldsymbol{\nu}$ are the vectors of end-of-period wealth, consumption, labor supply, and Lagrange multipliers, respectively. These vectors are known from the steady-state equilibrium of the Bewley model. We also define $\boldsymbol{I}$ as the $(N_{tot} \times N_{tot})$-identity matrix, $\boldsymbol{\Pi}$ as the transition matrix across histories, and $\boldsymbol{P}$ as the diagonal matrix having 1 on the diagonal at h if h is not credit-constrained (i.e., $h \in \mathcal{C}_{\mathcal{H}}$), and 0 otherwise. Defining $\circ$ as the Hadamard product, we have:

$$\boldsymbol{S} = \Pi \boldsymbol{S}, \tag{166}$$

$$\boldsymbol{S} \circ x + \boldsymbol{S} \circ \boldsymbol{a} = (1+r)\boldsymbol{\Pi}^\top (\boldsymbol{S} \circ \boldsymbol{a}) + \frac{1}{\chi \tilde{\tau}} l^{1/\varphi+1} \boldsymbol{S} \circ (\boldsymbol{y})^{\tilde{\tau}}, \tag{167}$$

$$(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{a} = \boldsymbol{0}_{N_{tot}}, \tag{168}$$

which correspond to the definition of history sizes (144), the individual budget constraint (161), the definition of credit-constrained histories (155), respectively.

Computing the ξs. Denoting by $\boldsymbol{D_x}$ the diagonal $(N_{tot} \times N_{tot})$-matrix with the N_{tot}-vector $\boldsymbol{x}$ on the diagonal, the Euler equation (10) becomes: $\boldsymbol{D}_{u'(\boldsymbol{c})}\boldsymbol{\xi}^E = \beta(1 + r)\boldsymbol{\Pi}^\top \boldsymbol{D}_{u'(\boldsymbol{c})}\boldsymbol{\xi}^E + \boldsymbol{\nu}$, implying that:

$$\boldsymbol{\xi}^E = \left[\left(\boldsymbol{I} - \beta(1+r)\boldsymbol{\Pi}^\top\right)\boldsymbol{D}_{u'(\boldsymbol{c})}\right]^{-1}\boldsymbol{\nu}. \tag{169}$$

Finding the Constraints on the Social Weights ω. We now construct the constraints that the social weights $(\omega^f)_{f=1,\ldots,F}$ must fulfill for the steady-state allocation to be optimal for given values of the planner's instruments.

We first define the F-vector of social weights we are looking for: $\boldsymbol{\omega}^F = (\omega^f)_{f=1,\ldots,F}$. We will show that there are two vectors $\hat{\boldsymbol{L}}_1, \hat{\boldsymbol{L}}_2$ such that all the FOCs of the planner are fulfilled when $\hat{\boldsymbol{L}}_1\boldsymbol{\omega}^F = 0 = \hat{\boldsymbol{L}}_2\boldsymbol{\omega}^F$. Together with the normalization constraint ($\sum^f \omega^f = 1$), this will impose three constraints on $\boldsymbol{\omega}^F$, explaining why the weights are exactly identified in our quantitative exercise of Section 5. Similarly to $\boldsymbol{\omega}^F$, $\boldsymbol{m}^F := (m^1, \ldots, m^F)$ is the F-vector of type shares. For simplifying algebra, we define the $N_{tot} \times F$-matrix $\boldsymbol{R}_0$ that maps a F-vector into an N_{tot}-vector (recall that $\boldsymbol{1}_{N^f_{tot}} \in \mathbb{R}^{N^f_{tot}}$ is a N^f_{tot}-vector of 1):

$$\boldsymbol{R}_0 := \begin{bmatrix} \boldsymbol{1}_{N^1_{tot}} & 0 & 0 & 0 \\ 0 & \boldsymbol{1}_{N^2_{tot}} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \boldsymbol{1}_{N^F_{tot}} \end{bmatrix},$$

and the $N_{tot} \times F$-matrix $\boldsymbol{R}_1 := \boldsymbol{D_S}\boldsymbol{R}_0\boldsymbol{D}_{\boldsymbol{m}^F}$ that maps a F-vector into an N_{tot}-vector, but where history sizes and types shares have been accounted for. To obtain dimensions compatible with other vectors and matrices, we define $\boldsymbol{\omega} := \boldsymbol{R}_0\boldsymbol{\omega}^F$ and:

$$\bar{\boldsymbol{\omega}} = \boldsymbol{R}_1\boldsymbol{\omega}^F. \tag{170}$$

We define the following quantities (with history-size accounted for): $\bar{\boldsymbol{\lambda}} := \boldsymbol{S} \circ \boldsymbol{\lambda}$, $\bar{\boldsymbol{\psi}} := \boldsymbol{S} \circ \hat{\boldsymbol{\psi}}$, $\boldsymbol{S} \circ \tilde{\boldsymbol{\lambda}} := \boldsymbol{\Pi}\bar{\boldsymbol{\lambda}}$, $\bar{\boldsymbol{\Pi}} := \boldsymbol{S} \circ \boldsymbol{\Pi}^\top \circ (\boldsymbol{1}_{N_{tot}}./\boldsymbol{S})$ (./ being the element-wise division).

With these definitions, the definition of ψ and the planner's FOCs (165) become:

$$\bar{\boldsymbol{\psi}} = \boldsymbol{D}_{\boldsymbol{\xi}^u \circ u'(\boldsymbol{x})} \bar{\boldsymbol{\omega}} - \boldsymbol{D}_{\boldsymbol{\xi}^E \circ u''(\boldsymbol{x})} \left(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}^\top \right) \bar{\boldsymbol{\lambda}} - \boldsymbol{S}\mu, \tag{171}$$

$$\frac{1}{\chi \tilde{\tau}} (1/\varphi + 1)\, l^{1/\varphi} \left(\boldsymbol{y}^{\tilde{\tau}} \right)^\top \bar{\boldsymbol{\psi}} = \mu \boldsymbol{S}^\top \left(\frac{l^{1/\varphi}}{\chi} \boldsymbol{y}^{\tilde{\tau}} - F_L \boldsymbol{y}^{1+\frac{\tilde{\tau}}{1/\varphi+1}} \right), \tag{172}$$

$$0 = \tilde{\boldsymbol{a}}^\top \bar{\boldsymbol{\psi}} + (\boldsymbol{\xi}^E \circ u'(\boldsymbol{x}))^\top (\boldsymbol{\Pi}^\top \bar{\boldsymbol{\lambda}}), \tag{173}$$

$$\frac{l^{1/\varphi+1}}{\chi \tilde{\tau}} \left(\boldsymbol{y}^{\tilde{\tau}} \circ \left(\log \boldsymbol{y} - \frac{1}{\tilde{\tau}} \mathbf{1}_{NF} \right) \right)^\top \bar{\boldsymbol{\psi}} = \frac{\mu l}{1/\varphi + 1} (\boldsymbol{S} \circ \log \boldsymbol{y})^\top \left(\frac{l^{1/\varphi}}{\chi} \boldsymbol{y}^{\tilde{\tau}} - F_L \boldsymbol{y}^{\frac{1/\varphi+1+\tilde{\tau}}{1/\varphi+1}} \right), \tag{174}$$

as well as the FOCs for savings for unconstrained and constrained histories: $\boldsymbol{P}(\boldsymbol{I} - \beta(1+r)\bar{\boldsymbol{\Pi}}\bar{\boldsymbol{\psi}}) = 0$ and $(\boldsymbol{I} - \boldsymbol{P})\bar{\boldsymbol{\lambda}} = 0$, respectively – where we denote by $\boldsymbol{I}$ the $N_{tot} \times N_{tot}$-identity matrix. Summing the two previous equations and using the definition (171) of $\bar{\boldsymbol{\psi}}$ yields:

$$\bar{\boldsymbol{\lambda}} = \boldsymbol{M}_2 \bar{\boldsymbol{\omega}} + \mu \boldsymbol{V}_1, \tag{175}$$

where $\boldsymbol{M}_1 := \boldsymbol{P}(\boldsymbol{I} - \beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{D}_{\boldsymbol{\xi}^E \circ u''(\boldsymbol{x})}(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}^\top) + \boldsymbol{I} - \boldsymbol{P}$, $\boldsymbol{M}_2 := \boldsymbol{M}_1^{-1}\boldsymbol{P}(\boldsymbol{I} - \beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{D}_{\boldsymbol{\xi}^0 \circ u'(\boldsymbol{x})}$, and $\boldsymbol{V}_1 := -\boldsymbol{M}_1^{-1}\boldsymbol{P}(\boldsymbol{I} - \beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{S}$. With (171), we now get:

$$\bar{\boldsymbol{\psi}} = \boldsymbol{M}_3 \bar{\boldsymbol{\omega}} + \mu \boldsymbol{V}_2, \tag{176}$$

where $\boldsymbol{M}_3 := \boldsymbol{D}_{\boldsymbol{\xi}^0 \circ u'(\boldsymbol{x})} - \boldsymbol{D}_{\boldsymbol{\xi}^E \circ u''(\boldsymbol{x})}(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}^\top)\boldsymbol{M}_2$ and $\boldsymbol{V}_2 := -\boldsymbol{D}_{\boldsymbol{\xi}^E \circ u''(\boldsymbol{x})}(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}^\top \boldsymbol{V}_1) - \boldsymbol{S}$. Substituting equation (176) into FOC (172), we obtain:

$$\mu = \boldsymbol{L}_0^\top \bar{\boldsymbol{\omega}}, \tag{177}$$

where $C_1 := \boldsymbol{S}^\top(\frac{l^{1/\varphi}}{\chi}\boldsymbol{y}^{\tilde{\tau}} - F_L \boldsymbol{y}^{1+\frac{\tilde{\tau}}{1/\varphi+1}}) - \frac{1}{\chi\tilde{\tau}}(1/\varphi + 1)l^{1/\varphi}(\boldsymbol{y}^{\tilde{\tau}})^\top \boldsymbol{V}_2 \in \mathbb{R}$ and $\boldsymbol{L}_0^\top := \frac{1+1/\varphi}{\chi\tilde{\tau}} l^{1/\varphi}(\boldsymbol{y}^{\tilde{\tau}})^\top \boldsymbol{M}_3 / C_1$ is a N_{tot}-row vector. We can substitute μ using (177) into relationships (175) and (176) to express $\bar{\boldsymbol{\psi}}$ and $\bar{\boldsymbol{\lambda}}$ as a function of $\boldsymbol{\omega}$ only:

$$\bar{\boldsymbol{\psi}} = \boldsymbol{M}_4 \bar{\boldsymbol{\omega}} \text{ and } \bar{\boldsymbol{\lambda}} = \boldsymbol{M}_5 \bar{\boldsymbol{\omega}}, \tag{178}$$

where $\boldsymbol{M}_4 := \boldsymbol{M}_3 + \boldsymbol{V}_2 \boldsymbol{L}_0^\top$ and $\boldsymbol{M}_5 := \boldsymbol{M}_2 + \boldsymbol{V}_1 \boldsymbol{L}_0^\top$. We have expressed Lagrange multipliers has a function of social weights and the two FOCs (173) and (174) are remaining will imply two constraints on $\bar{\boldsymbol{\omega}}$. Using (178) with (173) and (174) yields:

$$0 = \boldsymbol{L}_1^\top \bar{\boldsymbol{\omega}} = \boldsymbol{L}_2^\top \bar{\boldsymbol{\omega}}, \tag{179}$$

$$\text{where: } \boldsymbol{L}_1^\top := \left(\tilde{\boldsymbol{a}}^\top \boldsymbol{M}_4 + \text{where: } (\boldsymbol{\xi}^E \circ u'(\boldsymbol{x}))^\top \boldsymbol{\Pi}^\top \boldsymbol{M}_5 \right),$$

$$\boldsymbol{L}_2^\top := \frac{l^{1/\varphi+1}}{\chi\tilde{\tau}} (\boldsymbol{y}^{\tilde{\tau}} \circ (\log \boldsymbol{y} - \frac{1}{\tilde{\tau}} \boldsymbol{1}_{N_{tot}}))^\top \boldsymbol{M}_4 - \frac{l(\boldsymbol{S} \circ \log \boldsymbol{y})^\top}{1/\varphi + 1} (\frac{l^{1/\varphi}}{\chi} \boldsymbol{y}^{\tilde{\tau}} - F_L \boldsymbol{y}^{\frac{1/\varphi+1+\tilde{\tau}}{1/\varphi+1}}) \boldsymbol{L}_0^\top.$$

A third constrain is the normalization of social weights to 1: $\boldsymbol{1}_{N_{tot}} \bar{\boldsymbol{\omega}} = 1$. With (170) and (179), we obtain: $\boldsymbol{M}_6 \boldsymbol{\omega}^F = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top$, where $\boldsymbol{M}_6 = \begin{bmatrix} \boldsymbol{L}_1 & \boldsymbol{L}_2 & \boldsymbol{1}_{N_{tot}} \end{bmatrix}^\top \boldsymbol{R}_1$ is a $3 \times F$-matrix. If $F = 3$, $\boldsymbol{M}_6$ is a square matrix, which is generically invertible. We deduce:

$$\boldsymbol{\omega}^F = \boldsymbol{M}_6^{-1} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top. \tag{180}$$

H Accuracy of the simulation

We compare the simulation outcomes implied the refined truncation method and the Reiter's method, which is a benchmark simulation method in the literature. For the truncation, we consider a uniform truncation length $N = 2$ and refined truncation lengths of 10 for all productivity levels The two methods use perturbation techniques around the same steady-state – which is the one of Section 5 implied by the calibration of Table 2.

In both methods, the fiscal instruments $(\tau^K, B, \kappa, \tau)$ are assumed to follow simple fiscal rules (the Reiter method does not allow one to solve Ramsey policies). After a public spending shock, we assume that both capital tax (τ^K) and progressivity (τ) remain constant, while the labor tax parameter κ (recall the labor tax increases with κ) adjusts to stabilize public debt B, following a rule à la Bohn (1998): $\kappa_t - \kappa_{ss} = -c^B(B_t - B_{ss})$, where κ_{ss} and B_{ss} are the steady-state values of κ and B, and $c^B > 0$ is set to $c_B = 0.1$, which ensures debt stability. We have also considered other fiscal rules to check that it does not affect the comparison outcomes. Finally, we consider different persistences of the public spending shock ρ_G. The simulation outcomes are plotted in Figures 6 and 7 for $\rho_G = 0.95$ and $\rho_G = 0.1$, where we report: public spending G, aggregate consumption C, investment I (the three component of aggregate demand), public debt B (all four in proportional deviation), and labor tax parameter κ (in level deviation).

We also report in Table 11, the mean and maximum absolute differences for these variables between the two simulation methods. We only focus on the case $\rho_G = 0.95$, which generates the largest differences.

Overall, we find that the truncation method provides a good approximation of the

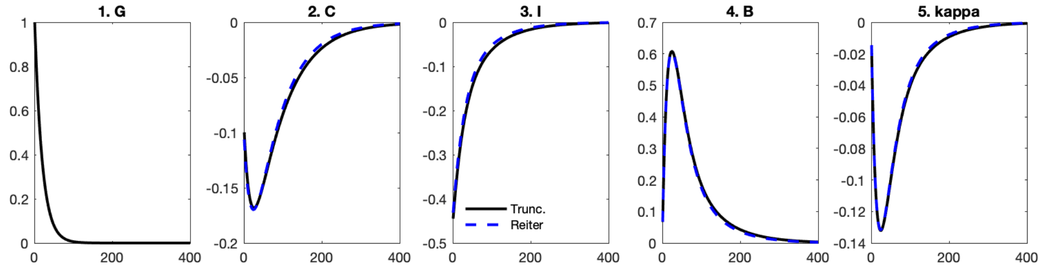

Figure 6: Simulation outcomes for $\rho_G = 0.95$. See text for details.

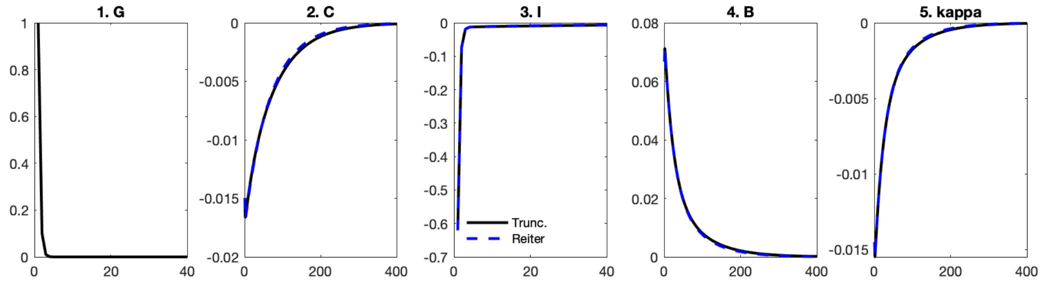

Figure 7: Simulation outcomes for $\rho_G = 0.1$. See text for details.

	Consumption C	Investment I	Public debt B	Labor tax par. κ
Mean abs. diff.	$1.18 \cdot 10^{-4}$	$2.35 \cdot 10^{-4}$	$2.06 \cdot 10^{-4}$	$4.48 \cdot 10^{-5}$
Max abs. diff.	$2.76 \cdot 10^{-4}$	$7.64 \cdot 10^{-4}$	$4.91 \cdot 10^{-4}$	$1.07 \cdot 10^{-4}$

Table 11: Mean and maximum absolute differences in simulations between the Reiter and the truncation methods ($\rho_G = 0.95$).

model dynamics, which confirms the results of LeGrand and Ragot (2022a) and LeGrand and Ragot (2022b) in different environments. The reason for this similarity is that the within-history time-varying heterogeneity has a second order effect on the dynamics compared to the between-history time-varying heterogeneity.

I Alternative Fiscal system

I.1 Model specification

We consider the same model as in Sections 2 and 4 but with a different fiscal system. Instead of using HSV to model progressivity, we consider a combination of a linear labor tax τ_t^L and of a lump-sum transfer (as in Dyrda and Pedroni, 2022 among others). The post-tax wage (4) becomes: $w_t := (1-\tau_t^L)\tilde{w}_t$, while the individual budget constraint (8) is:

$$c_{i,t}^f + a_{i,t}^f = (1+r_t)a_{i,t-1}^f + y_{i,t}^f l_{i,t}^f w_t + T_t. \tag{181}$$

The agent's program (7)–(9) is similar, but with equation (181) for the budget constraint. The governmental budget constraint (3) becomes:

$$G_t + T_t + (1+\tilde{r}_t)B_{t-1} \leq \tau_t^K \tilde{r}_t(B_{t-1}+K_{t-1}) + \tau_t^L \tilde{w}_t L_t + B_t, \tag{182}$$

where the labor supply is $L_t = (\chi w_t)^\varphi \sum_{f=1}^F m^f \sum_{y\in\mathcal{Y}} S_y y^{\varphi+1}$. Equivalently to (6), we can use the CRS property of the production function to express the governmental budget constraint using post tax prices:

$$G_t + T_t + r_t A_{t-1} + \chi^\varphi w_t^{\varphi+1} \sum_{f=1}^F m^f \sum_{y\in\mathcal{Y}} S_y y^{\varphi+1} = F(A_{t-1}-B_{t-1}, L_t) + B_t - B_{t-1}, \tag{183}$$

where $A_t = \sum_{f=1}^F m^f \int_i a_{i,t}^f \ell(di)$ is the aggregate savings. The Ramsey program (50)–(55) keeps the same structure but becomes with previous constraints:

$$\max_{\left(r_t,w_t,\tau_t,B_t,K_t,L_t,(a_{i,t}^f,c_{i,t}^f,l_{i,t}^f,\nu_{i,t}^f)_{i,f}\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^\infty \beta^t \sum_{f=1}^F \omega^f m^f \int_i u(x_{i,t}^f)\ell(di)\right], \tag{184}$$

subject to the governmental budget constraint (183), and for all f and i: the individual budget constraints (181), the credit constraints (54) and the Euler equations (53) (the last two constraints being unchanged).

I.2 Model Solution

The FOCs of the planner are computed with respect to public debt B_t, savings $a_{i,t}^f$, interest rate r_t, wage w_t, and transfer T_t. The FOCs wrt $a_{i,t}^f$, B_t, and r_t are similar to the ones in the HSV case and correspond to equations (58), (59), and (61), respectively. The FOC (61) wrt the labor tax parameter κ_t is replaced by the FOC wrt w_t:

$$0 = \sum_{f=1}^F m^f \int_i \hat{\psi}_{i,t}^f l_{i,t}^f y_{i,t}^f \ell(di) - \mu_t \varphi L_t \left(1 - \frac{F_{L,t}}{w_t}\right), \tag{185}$$

where $\hat{\psi}^f_{i,t}$ is defined in (57). The FOC (63) wrt progressivity τ_t has no meaning in this setup. The last FOC is the one wrt T_t:

$$0 = \sum_{f=1}^{F} m^f \int_j \hat{\psi}^f_{i,t} \ell(dj). \tag{186}$$

We solve the model using the truncation method. We thus construct a matrix representation of the model as in Section G.3. Equations (166), (168), and (169) still hold, but the budget constraint (167) becomes $\boldsymbol{S} \circ \boldsymbol{x} + \boldsymbol{S} \circ \boldsymbol{a} = (1+r)\Pi^\top (\boldsymbol{S} \circ \boldsymbol{a}) + \frac{\chi^\varphi w^{\varphi+1}}{1+\varphi} \boldsymbol{S} \circ (\boldsymbol{y})^{\varphi+1} + T \mathbf{1}_{N_{tot}}$.

The constraint $\boldsymbol{L}_1^\top \bar{\boldsymbol{\omega}} = 0$ of equation (179) is still valid, as well as the definition of L_1. Equations (175)–(178) and related vectors and matrices are also valid, but with $\tilde{\tau} = 1 + \varphi$ and $l = (\chi w)^\varphi$. For instance, we now have: $C_1 := (w - F_L)\boldsymbol{S}^\top \boldsymbol{y}^{1+\varphi} - \frac{w}{\varphi}(\boldsymbol{y}^{1+\varphi})^\top \boldsymbol{V}_2$. However, the constraint $\boldsymbol{L}_2^\top \bar{\boldsymbol{\omega}} = 0$ in (179) does not hold anymore as it is related to the progressivity $\tilde{\tau}$, which is not a planner's instrument anymore. This constraint is replaced by the one coming from the FOC on the transfer T_t, which implies: $\mathbf{1}_{N_{tot}}^\top \hat{\boldsymbol{\psi}} = 0$, or using (178), $\hat{\boldsymbol{L}}_2^\top \bar{\boldsymbol{\omega}} = 0$ with $\hat{\boldsymbol{L}}_2^\top = \mathbf{1}_{N_{tot}}^\top \boldsymbol{M}_4$. With $\hat{\boldsymbol{M}}_6 = \begin{bmatrix} \boldsymbol{L}_1 & \hat{\boldsymbol{L}}_2 & \mathbf{1}_{N_{tot}} \end{bmatrix}^\top \boldsymbol{R}_1$, the weight expression (180) becomes:

$$\boldsymbol{\omega}^F = \boldsymbol{M}_6^{-1} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top. \tag{187}$$

I.3 Model Calibration

Regarding the fiscal policy, we target a lump-sum transfer equal to 8% of GDP, which is consistent with Trabandt and Uhlig (2011). The linear labor tax is set to $\tau^L = 35.5\%$ to generate a public spending to GDP equal to 17%. The rest of the calibration (in particular the income process and all preference and technology parameters) are identical to the one of the baseline model in Table 2. The new elements of the calibration are in Table 12.

Parameter	Description	Value
Tax system		
τ^K	Capital tax	36%
τ^L	Scaling of labor tax	0.75
T/Y	Lump-sum transfers	8%
B/Y	Public debt	64%
G/Y	Public spending	17%

Table 12: Parameter values in the baseline calibration for the model with an affine tax structure. See text for descriptions and targets.

We solve the model using the refined truncation method with a uniform truncation length of 2 and a refined length of 10 for all productivity levels, as in Section 5.2. The dynamics of the fiscal system after a public spending shock with the same NPV but different persistences is reported in Figure 8 – similar to Figure 1 in the main text. The black solid lines correspond to persistence $\rho_G = 0.1$, and the blue dashed lines correspond to persistence $\rho_G = 0.99$. The following variables are reported: public spending, G; value of public resources, μ; level of labor tax, κ; progressivity of labor tax, τ; capital tax, τ^k; and public debt, B.

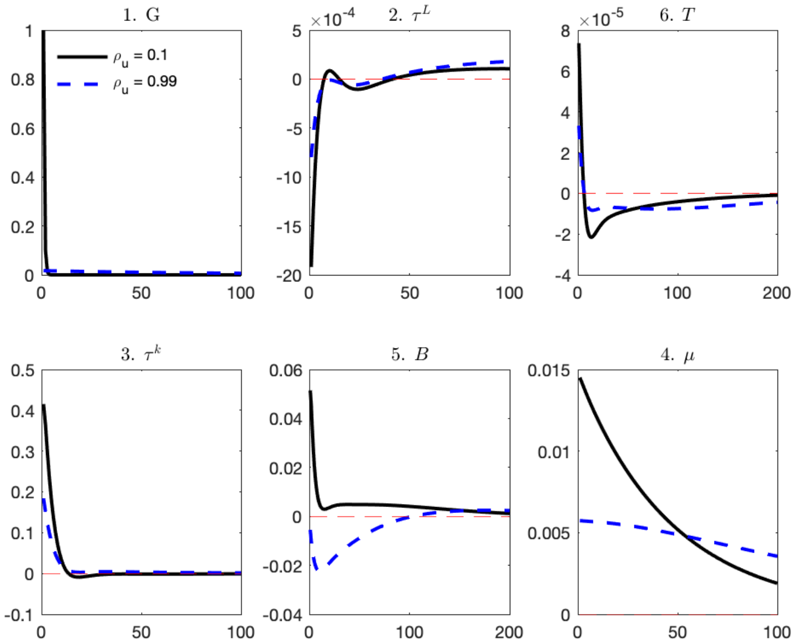

Figure 8: Dynamics of selected variables for two public spending shocks with different persistences and the same NPV in the model with an affine tax system. G—public spending; μ—value of public resources; κ—level of labor tax; τ—progressivity of labor tax; τ^k—capital tax; B—public debt. The black solid lines correspond to persistence $\rho_G = 0.1$, and the blue dashed lines correspond to persistence $\rho_G = 0.99$. G is in percent of GDP, B is in proportional deviations, and other variables are in level deviations.

In Figure 8, we can observe that when the persistence is low ($\rho^u = 0.1$), the public debt decreases on impact, while when the persistence is high ($\rho^u = 0.99$), the opposite holds and the public debt decreases on impact. In the two cases, the labor tax barely reacts and the small movement on impact – of the order of magnitude of $-10^{-3}\%$ – quickly vanishes. The capital tax steeply increases on impact by almost half a percent, which is much larger than the labor tax reaction. Finally, the lump-sum transfer T_t increases on impact, which makes the fiscal system more progressive. These results are all consistent

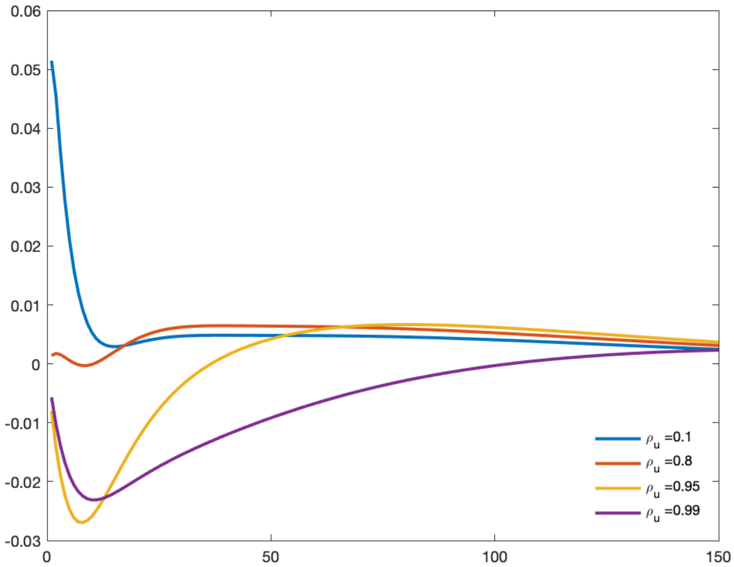

Figure 9: Comparison of optimal public debt dynamics in the model with an affine tax structure, for different persistences of the public spending shock (but the same NPV of public spending), in proportional deviation from steady-state value of public debt.

with the outcomes of the baseline model. We can also observe that the responses of public debt, capital tax and Lagrange multiplier of the governmental budget constraint, μ, are quantitatively very close with both tax systems.

Finally, we plot in Figure 9 the dynamic of public debt for different persistences of the public spending shock, while keeping the same NPV in all cases (as in Figure 3 of the main text). As already observed in Figures 1 and 8, the public debt responses are overall in the same ballpark for the two tax schemes. Furthermore, they are quantitatively very similar for the two redistribution schemes for very low and very large persistences. For interim values, the differences is slightly more marked and the transition from a positive to a negative public debt response on impact occurs for slightly lower persistence values when the tax system is affine than when it is progressive.

J Alternative SWF and period utility function system

J.1 Model specification

In this specification, agents have a separable utility function $U(c, l) = u(c) - v(l)$, and the fiscal system is the same as in the baseline case (HSV labor tax). All agents are ex-ante identical: the labor process is common to all agents.

In this setup, as we have learned from Section 3, and consistently with Chien and Wen (2023), that the Ramsey equilibrium does not exist when the planner is endowed with a utilitarian SWF. To address this concern, we assume that the SWF attributes weights to the period utility function that depend on the current productivity of the agent. Such a weight is denoted by $\omega(y_{i,t})$. The utilitarian case corresponds to $\omega(y) = 1$ for all y. This specification is used in an intertemporal setting by LeGrand et al. (2022), Dávila and Schaab (2022), and McKay and Wolf (2023) to deviate from the utilitarian case in a tractable way. Formally, the SWF that corresponds to the planner's aggregate welfare criterion can be expressed as

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_{i,t})(u(c_{i,t}) - v(l_{i,t}))\ell(di) \right]. \tag{188}$$

Using the same notation as in Section 2, the budget constraint of agents is $a_{i,t} + c_{i,t} = R_t a_{i,t-1} + w_t (y_{i,t} l_{i,t})^{\tau_t}$, while their FOCs with respect to consumption and labor can be written as: $u'(c_{i,t}) = \beta \mathbb{E}_t R_{t+1} u'(c_{i,t+1}) + \nu_{i,t}$ and $v'(l_{i,t}) = \tau_t w_t y_{i,t} (y_{i,t} l_{i,t})^{\tau_t - 1} u'(c_{i,t})$. We deduce that the Ramsey program can be written as follows:

$$\max_{(r_t, w_t, B_t, K_t, L_t, (a_{i,t}, c_{i,t}, l_{i,t}, \nu_{i,t})_i)_{t \geq 0}} W_0, \tag{189}$$

$$G_t + R_t B_{t-1} + (R_t - 1) K_{t-1} + w_t \int_i (y_{i,t} l_{i,t})^{\tau_t} \ell(di) = F(K_{t-1}, L_t) + B_t, \tag{190}$$

$$\text{for all } i \in \mathcal{I}\text{: } a_{i,t} + c_{i,t} = R_t a_{i,t-1} + w_t (y_{i,t} l_{i,t})^{\tau_t}, \tag{191}$$

$$a_{i,t} \geq -\bar{a}, \ \nu_{i,t}(a_{i,t} + \bar{a}) = 0, \ \nu_{i,t} \geq 0, \tag{192}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t R_{t+1} u'(c_{i,t+1}) + \nu_{i,t}, \tag{193}$$

$$v'(l_{i,t}) = \tau_t w_t y_{i,t} (y_{i,t} l_{i,t})^{\tau_t - 1} u'(c_{i,t}), \tag{194}$$

$$K_t + B_t = \int_i a_{i,t} \ell(di), \ L_t = \int_i y_{i,t} l_{i,t} \ell(di). \tag{195}$$

J.2 Model Solution

The model resolution is similar to the one of the baseline model, despite some minor differences. Because of the non-linear FOC (194) on the labor supply, we introduce a Lagrange multiplier on this FOC that we denote by $\beta^t \lambda_{l,i,t}$. We again introduce the quantities $\psi_{i,t}$ and $\hat{\psi}_{i,t}$ that we define as:

$$\psi_{i,t} := \omega_{i,t} u'(c_{i,t}) - (\lambda_{i,t} - R_t \lambda_{i,t-1}) u''(c_{i,t}) + \tau_t w_t \frac{\lambda_{l,i,t}}{l_{i,t}} (y_{i,t} l_{i,t})^{\tau_t} u''(c_{i,t}), \tag{196}$$

$$\hat{\psi}_{i,t} := \mu_t - \psi_{i,t}, \tag{197}$$

which are the parallels in this setup of the quantities defined in (56) and (57) in the baseline model, but which include the Lagrange multiplier on the labor supply FOC.

The FOCs of the Ramsey program (191) are computed with respect to public debt B_t, savings $a_{i,t}$, labor supply $a_{i,t}$ interest rate r_t, wage w_t, and transfer T_t. The FOCs wrt $a_{i,t}$, B_t, and r_t are similar to the ones in the main text and correspond to equations (58), (59), and (61), respectively. The FOCs (61) and (63) wrt the labor tax parameter κ_t and the progressivity τ_t are modified because of the separable period utility function. The new FOCs wrt w_t and τ_t are:

$$\int_j (y_{j,t} l_{j,t})^{\tau_t} \hat{\psi}_{j,t} \ell(dj) = \tau_t \int_j \frac{\lambda_{l,j,t}}{l_{j,t}} (y_{j,t} l_{j,t})^{\tau_t} u'(c_{j,t}) \ell(dj), \tag{198}$$

$$\int_j (y_{j,t} l_{j,t})^{\tau_t} \hat{\psi}_{j,t} \ell(dj) = \int_j \frac{\lambda_{l,j,t}}{l_{j,t}} (y_{j,t} l_{j,t})^{\tau_t} \left(1 + \tau_t \log(y_{j,t} l_{j,t})\right) u'(c_{j,t}) \ell(dj). \tag{199}$$

The FOC with respect to the labor supply $l_{i,t}$ is:

$$\begin{aligned} \omega_{i,t} v'(l_{i,t}) + \lambda_{l,i,} v''(l_{i,t}) = {} & \tau_t w_t y_{i,t} (y_{i,t} l_{i,t})^{\tau_t - 1} \hat{\psi}_{i,t} + \mu_t F_{L,t} y_{i,t} \\ & + \frac{\lambda_{l,j,t}}{l_{j,t}} \tau_t (\tau_t - 1) w_t (y_{i,t} l_{i,t})^{\tau_t - 1} u'(c_{i,t}). \end{aligned} \tag{200}$$

The model can be solved using the refined truncation method as explained in Section 5.2 for the baseline model and Section I.2 for the model with an affine tax structure. We skip the details here for saving space.

J.3 Calibration

The period is a quarter. The utility function is separable in labor $U(c, l) = u(c) - v(l)$, with $u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$ and $v(l) = \frac{1}{\chi} \frac{l^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}$. We set the inverse of IES to $\sigma = 2$, which is a standard value in the literature. The discount factor is set to $\beta = 0.99$.

The productivity process, common to all agents, follows a standard AR(1) process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, where $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. The parameters are $\rho_y = 0.993$ and $\sigma_y = 0.082$ to match the debt-to-GDP ratio with a relevant fiscal system. These parameters are close to those from a direct estimation of the productivity process on PSID data, which corresponds to $\rho_y = 0.9923$ and $\sigma_y = 0.0983$ (see Boppart et al., 2018 and Krueger et al., 2018). The productivity process is discretized into 7 states using the Rouwenhorst (1995) procedure.

The rest of the calibration is unchanged compared to the baseline model (technology parameters and fiscal system). Table 13 summarizes the model parameters that differ from the baseline calibration of Table 2.

Parameter	Description	Value
Preference and technology		
β	Discount factor	0.99
σ	Inverse of the IES	2
Productivity process		
ρ_y	Autocorrelation idio. income	0.993
σ_y	Standard dev. idio. income	0.082

Table 13: Parameter values in the calibration of the model with an alternative SWF. All other parameters are set to the same values as in the baseline calibration of Table 2.

We solve the model using the truncation method. We still use the inverse optimal taxation approach to estimate the social weights of the SWF, but the application differs because these are not ex-ante weights but weights on the period utility function depending on the productivity level. Because the number of productivity levels (here 7) is typically larger than the number of constraints imposed by the planner's FOCs (here 2 as in the baseline case), we cannot directly apply the same method as in Section 5.2. We follow Heathcote and Tsujiyama (2021) and assume that productivity weights admit a parametric representation that is quadratic in productivity levels. Formally, we assume that there exists two real parameters denoted θ_1 and θ_2, such that for all productivity levels y:

$$\log \omega(y) := \theta_1 \log y + \theta_2 (\log y)^2 .$$

With this specification, the parameters θ_1 and θ_2 are exactly identified with the two constraints from the Ramsey FOCs. Note that we do not impose any normalization of weights, as it would imply the addition of a parameter θ_0 (such that $\log \omega(y) := \theta_0 + \theta_1 \log y + \theta_2 (\log y)^2$), which would have no impact on the simulations. Our calibration implies $\theta_1 = 0.93$ and $\theta_2 = 0.33$. In an environment without savings, Heathcote and Tsujiyama (2021) estimated the relationship $\log \omega_y = \theta \log y$ and estimated $\theta = 0.517$.

The refined truncation length is set to $N = 20$, with a uniform truncation length of 2. Similarly to what we did for the baseline model in Appendix H, we check that the results are not sensitive to this choice and that it provides an accurate representation of the dynamics of the model.

J.4 Model Dynamics

Figure 10 plots the dynamics of the instrument for public spending shocks with two different persistences. The dynamics are qualitatively similar to thosein the baseline case. In particular, the public debt increases on impact for low persistence, while it decreases for large one. The response of the capital tax on impact is also much more sizable than the one of the labor tax. Finally, progressivity also increases. These three main results are the same as in the quantitative exercise with the baseline model of Section 5.3. In Figure 11, we plot the optimal debt dynamics for four values of the persistence of the public spending shock, with a normalization of the initial shock G_0 to generate the same NPV of public spending.

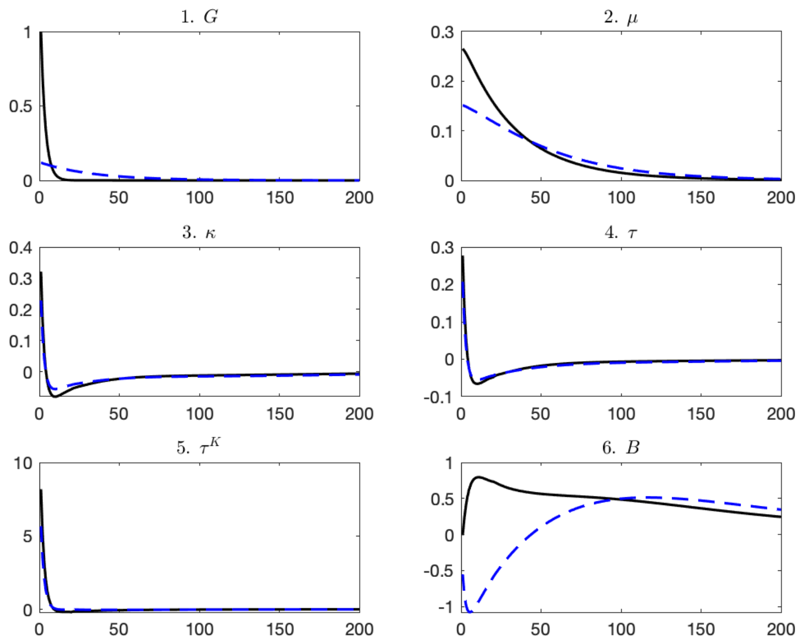

Figure 10: Dynamics of selected variables for two public spending shocks with different persistence but the same NPV. G—public spending; μ—value of public resources; κ—level of labor tax; τ—progressivity of labor tax; τ^k—capital tax; B—public debt. The black solid lines correspond to persistence $\rho_G = 0.7$, and the blue dashed lines correspond to persistence $\rho_G = 0.97$. G is in percent of GDP, B is in proportional deviations, and other variables are in level deviations.

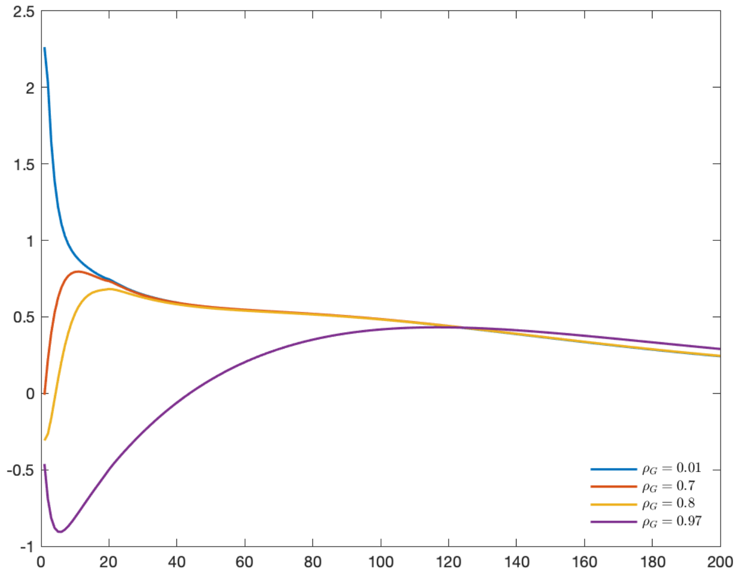

Figure 11: Comparison of optimal public debt dynamics for different persistences of the public spending shock (but the same NPV of public spending), in proportional deviation from steady-state value of public debt.

K Simulations with Other Shocks

We now present the simulations of the model with the TFP and discount factor shocks discussed in Section 5.4.3. We summarize the results using the same graph as in Figure 3, which shows the path of public debt for the different persistence values of the public spending shock.

Figure 12 plots the paths of public debt for different persistence values of the TFP shock. The initial value of the TFP shock is normalized so that the cumulative fall in TFP is identical in all cases.

Similarly, Figure 12 plots the paths of public debt for different persistence values of the discount factor shock. Again, the initial value of the shock is normalized for the average value of the discount factor to be the same in all cases.

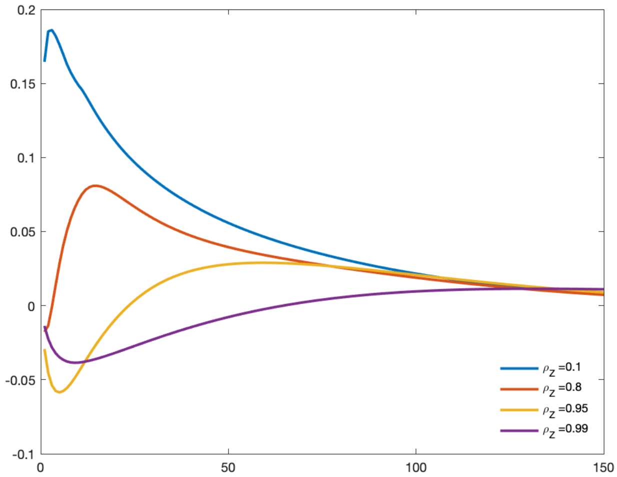

Figure 12: Comparison of optimal public debt dynamics for different persistences of the TFP (but the same average drop in TFP), in proportional deviation from steady-state value of public debt.

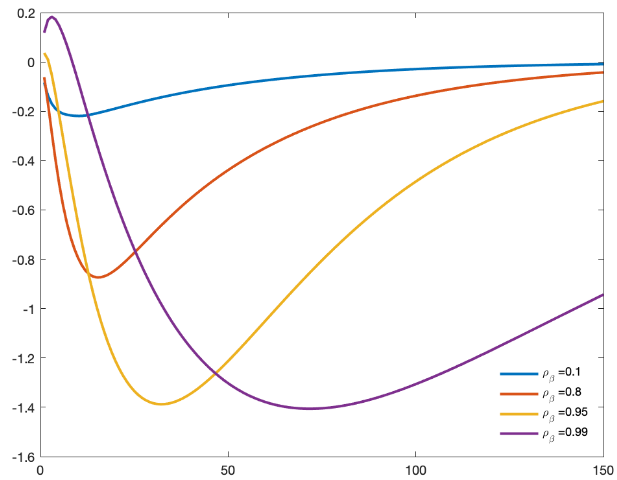

Figure 13: Comparison of optimal public debt dynamics for different persistences of the β-shock (but the same average value of the discount factor), in proportional deviation from steady-state value of public debt.