Bubbles and Self-fulfilling Crises*

Edouard Challe

Ecole Polytechnique, F-91128 Palaiseau, France

Email: edouard.challe@polytechnique.edu

Tel: +33 (0)1 69 33 30 11

Fax: +33 (0)1 69 33 34 27

Xavier Ragot

Banque de France, 39 rue des Petits Champs 75001 Paris, France

Email: xavier.ragot@banque-france.fr

Tel: +33 (0)1 42 92 49 18

January 14, 2010

*We received helpful feedback from seminar participants at the Universities of Cambridge, Paris-Dauphine, Paris X-Nanterre and Paris School of Economics, as well as from conference participants at the Paris Finance International Meeting (Paris, December 2005), the Theory and Methods of Macroeconomics Conference (Toulouse, January 2006), the Society for Economic Dynamics Conference (Vancouver, July 2006), and the Cycles and Crises workshop (Strasbourg, June 2008). We are particularly grateful to Jean-Pascal Benassy, Gilles Chemla and Rodolphe Dos Santos Ferreira for their comments and suggestions on earlier drafts of this paper. All remaining errors are ours.

Abstract: Financial crises are often associated with an endogenous credit reversal followed by a fall in asset prices and serious disruptions in the financial sector. To account for this sequence of events, this paper constructs a model where excessive risk-taking by investors leads to a bubble in asset prices, and where the supply of credit to these investors is endogenous. We show that the interplay between excessive risk-taking and the endogeneity of credit may give rise to multiple equilibria associated with different levels of lending, asset prices, and output. Stochastic equilibria lead, with positive probability, to an inefficient liquidity dry-up, a market crash, and widespread failures by borrowers. The possibility of multiple equilibria and self-fulfilling crises is shown to be related to the severity of the risk-shifting problem in the economy.

Keywords: Credit market imperfections; self-fulfilling expectations; financial crises.

JEL codes: G12; G33.

1 Introduction

The resurgence of financial crises over the past couple of decades or so, both in developed and developing countries, has sparked renewed interest in the potential sources of financial fragility and market imperfections from which they originate. While each crisis naturally had its own particular features, it is now widely agreed that many shared a common underlying pattern of destabilising credit and asset markets developments, with an initial lending and asset price boom abruptly ending in a market crash and major disorders in the financial sector. The subprime mortgage crisis that has disrupted worldwide financial markets from August 2007 on provides a particularly dramatic example of such a crash, as it followed a prolonged phase of sustained lending fostered by low interest rates, new financial instruments and the poor *ex ante* pricing of the downside risk associated with falls in house prices.[1] But the subprime mortgage crisis, as striking as it is due to the size of the losses involved, is only the latest and most emblematic example of a long series. Amongst OECD countries in the 1980s and early 1990s, such as Japan or the Scandinavian countries, financial crises were an integral part of a broader credit cycle whereby financial deregulation led to an increase in available credit, fuelled a period of overinvestment in real estate and stock markets, and led to high asset-price inflation. These events were then followed by a credit contraction and the bursting of the asset bubble, causing the actual or near bankruptcy of the financial institutions which had initially levered the asset investment.[2] A similar sequence of events has also been observed in a number of Asian and Latin American countries, where capital account liberalisation allowed large amounts of capital to flow in during the 1990s, with a similar effect of raising asset prices to unsustainable levels. This phase of overlending often ended in a brutal capital account reversal followed by a market crash and a banking crisis.[3]

An important theoretical issue,to date largely unanswered, is whether the credit turn-

[1] See Greenlaw *et al.* (2008), Demyanyk and Van Hemert (2008) and International Monetary Fund (2008) for descriptive accounts of the boom-bust cycle in subprime mortgage loans, as well as Bordo (2007) for a historical perspective on the crisis.

[2] See Borio, Kennedy and Prowse (1994) and Allen and Gale (1999, 2000), as well as the references therein, for a more detailed account of these events.

[3] See Calvo (1998), Kaminsky (1999) and Kaminsky and Rheinart (1998, 1999) for evidence on this sequence of events, often referred to as 'sudden stop'.

around that typically accompanies financial crises is the outcome of an autonomous, extrinsic reversal of expectations on the part of economic agents, or simply the natural outcome of accumulated macroeconomic imbalances or policy mistakes, i.e., the intrinsic fundamentals of the economy. For a time, the consensus was to interpret crises simply as the outcome of extraneous sunspots hitting the beliefs of investors, regardless of the underlying fundamental soundness of the economy. For example, early models of crises would emphasise the inherent instability of the banking system, whose provision of liquidity insurance made banks sensitive to self-fulfilling runs, as the ultimate source of vulnerability to crises.[4] In a similar vein, 'second-generation' models of currency crises would insist on the potential existence of multiple equilibria in models of exchange rate determination, where the defense of a pre-announced peg by the central bank is too costly to be fully credible.[5]

Although such expectational factors certainly play a rôle in triggering financial crises, theories based purely on self-fulfilling expectations clearly do not tell the full story. In virtually all the recent episodes briefly mentioned above, specific macroeconomic or structural sources of fragility preceded the actual occurrence of the crisis. For example, poor risk assessment by both mortgage loan originators and buyers of mortgage-backed securities played a central role in the subprime lending bubble (International Monetary Fund, 2008). The OECD financial crises of the late 1980s usually followed periods of loose monetary policy or poor exchange-rate management (e.g., Borio *et al.*, 1994). In emerging countries, the culprit was often to be found in the weakness of the banking sector due to poor financial regulation, as well as other factors such as unsustainable fiscal or exchange rate policies (Summers, 2000). Overall, the evidence from this latter group of countries indicates that factors of fundamental weakness explain only some of the probability of a crisis, suggesting that both fundamental and non-fundamental elements are at work in triggering financial crises (see Kaminsky, 1999, and the discussion in Chari and Kehoe, 2003).

The model of financial crises that we develop below aims to account for both the credit-asset price cycle typical of recent crises and the joint role of fundamental and nonfundametal factors in making crises possible. In so doing, we draw on Allen and Gale (2000), for whom financial crises are the natural outcome of credit relations where portfolio investors borrow to

[4] See Diamond and Dybvig (1983), as well as Chang and Velasco (2002) for an open-economy model.

[5] E.g., Obsfeld (1996) and Velasco (1996).

buy risky assets, and are protected against bad payoff outcomes by the use of debt contracts with limited liability. Investors' distorted incentives then lead them to overinvest in risky assets (i.e., a *risk-shifting* problem arises), whose price consequently rises to high levels (leading to an *asset bubble*), with the possibility that investors become bankrupt if asset payoffs turn out badly (a *financial crisis* occurs). Unlike Allen and Gale, however, who study the risk-shifting problem in isolation and thus make the partial-equilibrium assumption that the amount of funds available to investors is exogenous, we allow for endogenous variations in the supply of credit resulting from lenders' utility-maximising behaviour. We regard this alternative specification as not only more realistic, but also particularly relevant to our understanding of recent crises episodes, where the endogeneity of aggregate credit was frequently identified as being an important source of financial instability.[6]

Our results indicate that the interdependence between excessive risk-taking by investors and the elasticity of aggregate credit is indeed a serious cause of endogenous instability. First, we show that, under risk-shifting, the equilibrium return that lenders expect from lending to investors may be non-monotonic and *increase* with the aggregate quantity of loans, rather than *decrease* as standard marginal productivity arguments would suggest. The explanation is that investors' optimal portfolio composition typically changes as the amount of funds that is lent to them varies, i.e., the 'assets' and 'liabilities' sides of investors' balance-sheets are not independent. In certain circumstances, which we derive and explain in the paper, an increase in investors' liabilities may shift the composition of the portfolio in such a way as to raise the *ex ante* return on loans. When this 'portfolio composition' effect is strong enough, it may dominate the usual 'marginal productivity' effect, so that the expected return on loans increases with aggregate loans (at least for some range of total loans). This strategic complementarity naturally leads to the existence of multiple equilibria associated with different levels of aggregate lending, asset prices, and output. We relate the intensity of these strategic complementarities, and the resulting possibility of multiple equilibria, to the severity of the risk-shifting problem in the economy.

We then consider the case where multiple equilibria do exist, and where the selection of an equilibrium with low lending follows a 'sunspot', i.e., an extraneous signal of any

[6] See, for example, Edison, Luangaram and Miller (2000) for a contribution which is representative of this view.

ex ante probability on which agents coordinate their expectations. We show that such stochastic equilibria generate *self-fulfilling crises* with the following characteristics; i) lending to portfolio investors drops off as lenders choose to store, rather than lend, a large share of their endowment (credit contraction), ii) this causes a fall in investors' resources and a drop in their demand for fixed-supply assets, whose price consequently falls to low levels (market crash), and iii) this fall in prices forces into bankruptcy investors who had previously borrowed to buy assets, as the new value of their assets falls short of their liabilities (financial sector disruptions). In short, *weak fundamentals* make multiple equilibria possible, while *self-fulfilling expectations* trigger the actual occurrence of the crisis. We also provide a full welfare analysis of the self-fulfilling crisis model. Crises are shown to unambiguously decrease *ex ante* welfare, with a principal source of this welfare loss being the negative *wealth effects* of the crash on lenders' consumption.

Although our theory of financial crises draws on recent related contributions, it also differs from them in a number of respects. While Allen and Gale (2000) and Edison *et al.* (2000) both emphasise the interdependency between asset price movements and aggregate credit during crises, they do so in the framework of single-equilibrium models where crises are entirely explained by exogenous fundamentals. Building on the empirical results of Kaminsky (1999) discussed above, Chari and Kehoe (2003) account for crises which are unexplained by fundamental factors by relying on investors' 'herd behaviour' in an environment with heterogenous information; in contrast, our results are derived within a rational expectations framework where all investors share the same information about asset payoffs. Finally, within the class of multiple-equilibrium based theories, our framework differs from 'third generation' models of currency crises (e.g., Aghion, Bacchetta and Banerjee, 2001 and 2004) by focusing on the instability of aggregate credit, rather than the volatility of nominal exchange rates; it also differs from infinite-horizon models where self-fulfilling asset-price movements are the outcome of 'steady state indeterminacy', i.e., the multiplicity of converging perfect-foresight equilibrium paths (as in Challe, 2004, for example).[7]

[7] Caballero and Krishnamurthy (2006) offer a model of emerging country bubbles where the bursting of the bubble is associated with a capital flow reversal. In their model, the existence of bubbles is related to the relative scarcity of available stores of value (as in Tirole (1985)), while our bubbles owe their existence to agency problems in the financial sector leading to excessive risk-taking by investors.

The remainder of the paper is organised as follows. Section 2 introduces the model and derives its unique fundamental (i.e., first-best efficient) equilibrium. Section 3 shows how the interdependency between endogenous lending and the excessive risk-taking of portfolio investors may give rise to multiple equilibria associated with different levels of lending, asset prices, and output. Section 4 derives the stochastic equilibria of this economy (i.e., equilibria featuring self-fulfilling crises) and analyses their welfare properties. Section 5 tests the robustness of our results by relaxing several baseline assumptions, and Section 6 concludes. All the proofs of the stated propositions are presented in an Appendix.

2 The model

2.1 Timing and assets

There are two dates, 1 and 2, and three real assets, labelled *production*, *risky asset*, and *storage*. Production yields $f(x)$ units of the (all-purpose) good at date 2 for $x \geq 0$ units invested at date 1, where $f(.)$ is a twice continuously differentiable function satisfying $f'(x) > 0$, $f''(x) < 0$, $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$. Moreover, the following standard assumption is made to limit the curvature of $f(.)$, for all $x > 0$:

$$\eta(x) \equiv -xf''(x)/f'(x) < 1. \tag{1}$$

The risky asset is in fixed supply (normalised to 1); it is available for purchase at date 1 and delivers a terminal payoff R at date 2, where R is a random variable at date 1 that takes on the value R^h with probability $\pi \in (0,1]$, and 0 otherwise, at date 2. Although more general distributions for the fundamental uncertainty affecting the asset payoff can be envisaged, we choose this simple specification in order to focus on the extrinsic uncertainty generated by the presence of multiple equilibria. The market price of the risky asset at date 1, in terms of the good (which is taken as the numeraire), is denoted by P_1.

Storage yields $\tau y > 0$ units of goods at date 2 for y units invested at date 1. For expositional simplicity and with no loss of generality, it is assumed that when agents are exactly indifferent between storing and investing in other assets, then they do not store.[8]

[8] In theory, the level of storage should be indeterminate when the return on storage equals that on other assets, but it turns out that this never occurs in equilibrium. Thus, assuming from the onset that storage is

The interpretation of this menu of available assets is that the supply of the risky asset responds slowly to changes in its demand (for example, real estate), while that of the safe assets adjusts quickly, and we consider the way markets clear in the short run. There are several possible interpretations for the storage technology assumed here. It may reflect the possibility for agents to store wealth in the form of cash balances or government bonds; in the first case τ is just the inverse of the inflation rate, and in the second the inflation-adjusted government bonds rate. Alternatively, one can think of the model as representing a small open economy where domestic agents have access to the pool of world liquidity, which may also include foreign government bonds and high quality foreign corporate bonds.

Our baseline assumptions that the supply of risky assets is completely fixed while the supply of storage is fully flexible are admittedly extreme and simplistic. To check that our results do not hinge too much on these assumptions, Section 5.1 analyses a simple extension to the baseline model where both assumptions are relaxed; we there show that all our results continue to hold provided that the supply of the risky asset is sufficiently less flexible than that the safe asset and that the return on storage is not too responsive to the total amount stored.

2.2 Agents and market structure

The economy consists of four types of risk neutral agents in large numbers, all maximising terminal consumption.[9] There is a continuum of two-period lived *lenders* of mass 1 who consume at date 2 and receive an endowment e_1 at date 1 satisfying

$$e_1 > f'^{-1}(\tau) + \pi R^h/\tau. \tag{2}$$

As will become clear below, this technical assumption ensures that all the equilibria that we analyse in the paper correspond to interior solutions, i.e., where all three real assets are held in equilibrium.

Lenders face two-period lived *investors* and *entrepreneurs* with positive mass who enter the market at date 1 and consume at date 2. Neither of them receive any endowment.

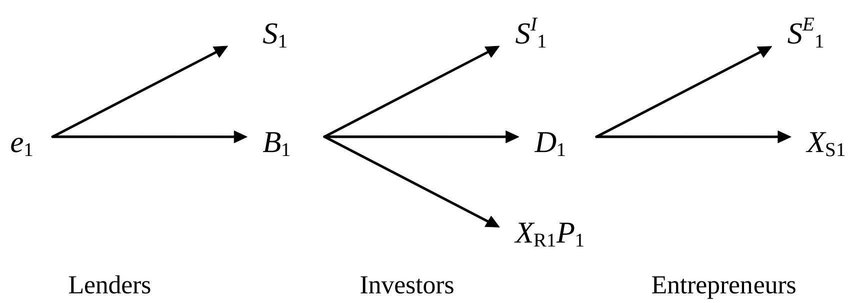
zero in case of equal returns allows us to avoid dealing with such virtual portfolios when deriving the optimal behaviour of individual agents.

[9]The paper focuses on the risk-neutral case, in which all results can be derived analytically. The risk-averse case is explored numerically as an extension to the baseline model in Section 5.2.

Finally, the stock of risky assets is initially held by a class of one-period lived *initial asset holders*, who sell them to investors at date 1 and then leave the market.

There is *market segmentation* (i.e., restrictions on agents' asset holdings) in the following two senses. First, only entrepreneurs have access to the production technology $f(.)$; since they have no wealth of their own, they borrow funds by issuing D_1 bonds at date 1. Second, only investors have the asset management ability necessary to trade corporate bonds and risky assets. Since lenders are excluded from these markets, they can only store or lend their funds to investors to finance date 2 consumption; denoting lenders' storage by S_1 and their loans to investors by B_1, we thus have $S_1 + B_1 \leq e_1$. Similarly, since entrepreneurs do not engage in security trading, they can only invest their borrowed funds into storage and productive investment; denoting by S_1^E and X_{S1} entrepreneurs' storage and productive investment, respectively, we have $S_1^E + X_{S1} \leq D_1$. These assumptions about market segmentation imply that the equilibrium at date 1 is partly *intermediated*, with lenders first entrusting investors with some of their savings (i.e., lending B_1), and then investors lending to entrepreneurs (i.e., buying D_1 corporate bonds), investing in risky assets (i.e., buying X_{R1} assets at price P_t), and possibly storing the rest, S_1^I (so that $X_{R1}P_1 + D_1 + S_1^I \leq B_1$). For ease of presentation and future reference, the flow of funds running from lenders to other agents at date 1 is summarised in Figure 1.

Figure 1: Flow of funds



As we shall establish below, in general equilibrium investors and entrepreneurs strictly prefer to invest all their borrowed resources where they hold a comparative advantage (asset trading and production, respectively) and thus never find it worthwhile to store. Thus, although we will have $S_1^I = S_1^E = 0$ in equilibrium (and hence $X_{S1} = D_1$ and $X_{R1}P_1 + X_{S1} =$

B_1), this will reflect agents' optimal portfolio choice, rather than exogenous restrictions on their access to the storage technology.

We think of our investors as being private, highly leveraged financial institutions that operate directly in the financial markets, such as investment banks and hedge funds. They may also include commercial banks or other leveraged intermediaries, to the extent that they engage in security trading as a secondary activity or hold loans whose recovery rate is tied to fluctuating asset prices (for example, collateralised mortgages). The key difference between such institutions and non-leveraged investors (like households or insurance companies) is that limited liability on the liability side coupled with market risk on the asset side may force the former into bankruptcy in case of bad asset performance, leaving lenders with the residual value of assets.[10] To allow for the possibility of investor default, we follow Allen and Gale (2000) in assuming that lenders and investors use simple debt contracts, where the contracted rate on these loans, r_1^l, cannot be conditional on the loan size or, due to asymmetric information, the investor's portfolio. As we show below, the use of debt contracts with limited liability causes lenders' and investors' incentives to be misaligned, with investors taking riskier asset positions than lenders would if they had direct access to all investment opportunities. Note that the distorting effect of debt financing (as opposed to equity financing) for value-maximising decisions, and the resulting excess risk-taking that may ensue, has been well understood at least since the work of Jensen and Meckling (1976). While we do not seek to provide a fully microfounded account of the use of debt contracts here, which would be well beyond the scope of this paper, we find it helpful to think of them as originating from a "double moral hasard" problem of the type analysed by Biais and Casamatta (1999), among others. Imagine, for example, that an investor's payoff depends not only on the riskiness of his chosen portfolio but also on his asset management effort, both of which are concealed to lenders. To elicit high effort, the efficient contract must reward the investor generously when the payoff is high. A simple debt contract fulfils this purpose (by letting the borrower capture all of the payoff in excess of the due debt repayments), even

[10] Leveraged investors played a central role in the run up to the subprime mortgage crisis. According to Greenlaw *et al.* (2008, p. 25), US and foreign-based leveraged intermediaries accounted for about two thirds of the total exposure to subprime mortgage risk. The growing share of risky assets held by leveraged investors in recent years is documented in International Monetary Fund (2008, ch. 2). See also Adrian and Shin (2007) for evidence on the procyclical behaviour of these intermediaries.

though it may lead the investor to hold a riskier portfolio that in the first-best case.[11]

Although risk shifting arises from the use of debt contracts in our model, it is worth stressing that other well-known market distortions are likely to generate similar incentive problems. For example, it is frequently argued that the *compensation schemes* enjoyed by money managers, often characterised by a convex reward structure, lead them to take excessively risky asset positions.[12] At the macroeconomic level, explicit or implicit *government guarantees* have also often been blamed for leading investors to select their portfolio on the basis of the upper end of the payoff distribution, in the expectation that any large loss incurred in the case of bad payoff outcomes will be socialised.[13] We thus think of the limited liability nature of debt contracts as one amongst a number of factors potentially leading to excessive risk taking by investors.

2.3 Fundamental equilibrium

In the intermediated economy described above, entrepreneurs are granted exclusive access to the production technology while only investors can trade risky assets and corporate bonds. Before analysing the resulting market outcome in more detail, it is useful to first derive the equilibrium that would prevail without these restrictions, i.e., if households were able to directly invest in all assets. The corresponding 'fundamental' equilibrium, in which prices and quantities are first-best efficient, will provide a natural benchmark against which the intermediated equilibrium can be compared.

In this equilibrium, households freely allocate their endowment e_1 across the three real assets available. Using the superscript F to index the fundamental equilibrium, households choose productive investment, X_{S1}^F, risky asset holdings, X_{R1}^F, and storage, S_1^F, so as to

[11] A related point is made by Barlevy (2008), who showed that simple debt contracts involving risk shifting may be optimal when lenders can not distinguish speculative investors from well-behaved entrepreneurs.

[12] See Chevalier and Ellison (1997) for an empirical study of how incentives affect risk taking by fund managers, and Palomino and Prat (2005), as well as the references therein, for models of investor risk taking under portfolio delegation.

[13] Explicit government guarantees include those enjoyed by capital inflows into some South East Asian countries prior to the 1997 crisis (see Corsetti *et al.*, 1999). Implicit guaranties also lead to expectations of bail out that can reasonably be qualified as rational. In the sole case of the subprime mortgage crisis, most distressed banks have received direct or indirect public support aimed at avoiding ex post bankrupcy.

maximise expected terminal consumption, taking the price of the risky asset, P_1^F, as given. The lenders' objective is thus:

$$\max E\left(\tau S_1^F + f\left(X_{S1}^F\right) + X_{R1}^F R\right)$$
$$\text{s.t. } X_{S1}^F + X_{R1}^F P_1^F + S_1^F \leq e_1,$$
$$X_{S1}^F, X_{R1}^F, S_1^F \geq 0,$$

were expectations are conditional on the information set at date 1. Substituting the first constraint into the objective and rearranging, the lenders' problem becomes:

$$\max e_1\tau + X_{R1}^F\left(\pi R^h - \tau P_1^F\right) + f\left(X_{S1}^F\right) - \tau X_{S1}^F. \tag{3}$$

From equation (3), no-arbitrage considerations imply that the fundamental value of the asset must be:

$$P_1^F = \pi R^h/\tau. \tag{4}$$

The return to storage, τ, is the opportunity cost of holding risky assets, and thus the rate at which expected dividend payments, πR^h, are discounted. Were the fundamental value of the risky assets to be greater than $\pi R^h/\tau$, then the gross return on trading assets, $\pi R^h/P_1^F$, would be lower than the storage return τ for all positive values of X_{R1}^F; no lender would be willing to buy the risky asset, which would drive its price down to zero and its expected return up to infinity. On the other hand, were P_1^F to be smaller than $\pi R^h/\tau$, then the gross return $\pi R^h/P_1^F$ would be higher than τ for all positive values of X_{R1}^F; lenders would all compete to buy the risky asset only and would bid up its price until $P_1^F \geq \pi R^h/\tau$. Thus, neither $P_1^F < \pi R^h/\tau$ nor $P_1^F > \pi R^h/\tau$ can be equilibrium situations. Then, choosing X_{S1}^F to maximise (3) gives:

$$X_{S1}^F = f'^{-1}(\tau). \tag{5}$$

For future reference and comparison with the intermediated equilibrium, we denote by B_1^F the total amount of funds invested in production and risky assets in the fundamental equilibrium. We have:

$$B_1^F = f'^{-1}(\tau) + \pi R^h/\tau, \tag{6}$$

while the implied fundamental level of storage, $S_1^F = e_1 - B_1^F$, is positive by assumption (2).

3 Endogenous lending and multiple equilibria

This Section computes the intermediated equilibrium, i.e., where households no longer have direct access to the markets for risky assets and corporate bonds. First, entrepreneurs' and investors' optimal decisions are used to compute the market-clearing asset-price vector (P_1,r_1) *conditional* on aggregate lending, B_1 (Section 3.1). Second, lenders' *ex ante* return on their loans to investors is derived, given this price vector and the possibility that investors default at date 2 (Section 3.2). Third, the loan return curve, and the implied lenders' choices, determine aggregate lending and asset prices in equilibrium (Section 3.3). Finally, the main properties of the intermediated equilibrium are discussed (Sections 3.4 and 3.5).

3.1 Market clearing

Corporate investment and bond rate. In the intermediated equilibrium, entrepreneurs borrow D_1 unit of funds at date 1 and turn these funds into real investment, X_{S1}, and storage, S_1^E (see Figure 1). They thus solve:

$$\begin{aligned} &\max f\left(X_{S1}\right) + \tau S_1^E - r_1 D_1 \\ &= \max f\left(X_{S1}\right) - r_1 X_{S1} + S_1^E\left(\tau - r_1\right), \\ &\text{s.t. } X_{S1}, S_1^E \geq 0, \end{aligned}$$

where r_1 is the gross interest rate on corporate bonds. No-arbitrage considerations indicate that we must have that $r_1 \geq \tau$ and thus $S^e\left(\tau - r_1\right) = 0$. If $r_1 < \tau$ then entrepreneurs would be willing to issue infinitely many bonds and store the proceeds; they would hit the limit of available funds in the economy (since the aggregate endowment, e_1, is finite), and from this point would compete for loans until $r_1 \geq \tau$. Then, if $r_1 \geq \tau$, the return on storage is strictly less than, or equal to, the corporate bond rate and entrepreneurs choose $S_1^E = 0$ (recall that agents do not store when the net return on doing so is zero). Thus, the solution to entrepreneurs' portfolio choice is such that $D_1 = X_{S1}$ and

$$f'\left(X_{S1}\right) = r_1 \geq \tau. \tag{7}$$

Contracted loan rate. Investors borrow B_1 (≥ 0) from lenders, which they use to buy X_{S1} corporate bonds, X_{R1} risky asset (at price P_1), and possibly to store the remainder, S_1^I. The use of debt contracts with limited liability allows investors to default, and earn 0, when their total payoff at date 2, $r_1 X_{S1} + R X_{R1} + \tau S_1^I$, is less than the amount owed to lenders, $r_1^l B_1$. Their terminal consumption, conditional on the risky asset's payoff R, is thus:[14]

$$\max\left[r_1 X_{S1} + R X_{R1} + \tau S_1^I - r_1^l B_1, 0\right],$$
$$\text{s.t. } X_{S1} + P_1 X_{R1} + S_1^I \leq B_1,$$
$$X_{S1}, X_{R1}, S_{R1}^I \geq 0.$$

Using the first constraint and rearranging, we can write investors' consumption as:

$$\max\left[X_{S1}\left(r_1 - r_1^l\right) + X_{R1}\left(R - r_1^l P_1\right) + S_1^I\left(\tau - r_1^l\right), 0\right].$$

A no-arbitrage argument similar to that used to characterise the behaviour of entrepreneurs allows us to infer that $r_1^l \geq \tau$ (otherwise investors would want to borrow an unlimited amount of funds and store them), and thus $S_1^I = 0$. It must also be the case that the contracted rate on loans between lenders and investors, r_1^l, be equal to the interest rate on corporate bonds, r_1. If $r_1 > r_1^l$, then investors would want to borrow an unlimited amount of funds from lenders and use them to buy corporate bonds; they would then reach the finite limit of available funds, and from then on compete for loans until $r_1 = r_1^l$. If $r_1 < r_1^l$ then investors' loan demand would be zero, so that the return on corporate bonds would be $r_1 = f'(0) = \infty$, a contradiction. Thus, any equilibrium in the markets for loans and corporate bonds must satisfy $r_1^l = r_1 = f'(X_{S1})$. At this loan rate, perfect competition amongst investors drives down the net return on trading corporate bonds to zero.

Asset prices and interest rate. Since $X_{S1}\left(r_1 - r_1^l\right) + S_1^I\left(\tau - r_1^l\right) = 0$, investors' terminal consumption is simply $\max\left[X_{R1}\left(R - r_1 P_1\right), 0\right]$. Because $X_{R1}\left(0 - r_1 P_1\right) < 0$ for all $P_1 > 0$, investors default on loans when the asset payoff is 0, and this occurs with probability $1 - \pi$. Their expected date 2 consumption is thus $\pi X_{R1}\left(R^h - r_1 P_1\right)$, provided they do not default when the asset payoff is R^h (i.e., provided $X_{R1}\left(R^h - r_1 P_1\right)$ is non-negative, as is always the

[14] Our formulation for investors' objective reflects the simplifying assumption that they have no equity. It can be shown that our results are unchanged provided that investors' equity is sufficiently small, while the intermediated equilibrium is identical to the fundamental one when the amount of equity is large. This is why we interpret our investors as highly-leveraged intermediaries –see our our discussion in Section 2.2.

case in equilibrium). Given their objective of maximising expected terminal consumption, market clearing for the risky asset implies that its equilibrium price is:

$$P_1 = R^h / r_1. \tag{8}$$

Were the price of the asset to be lower (higher) than R^h/r_1, then $R^h - r_1 P_1$ would be positive (negative) for all positive values of X_{R1} and investors would want to buy infinitely many (zero) risky assets. Notice from (8) that investors' consumption when $R = R^h$ is $X_{R1}\left(R^h - r_1 P_1\right) = 0$. The reason for this is intuitive: because markets are competitive, investors must make zero expected profits on trading risky assets. Since they earn zero when $R = 0$ and they default, they must also earn zero when $R = R^h$, which is exactly ensured by the equilibrium price in (8). Thus, in equilibrium the terminal consumption of investors is zero under both possible values of R at date 2.

Using equation (8) and the fact that in equilibrium $X_{R1} = 1$, $S_1^I = S_1^E = 0$ and $r_1 = f'(X_{S1})$, we have $r_1 = f'(B_1 - P_1)$. Market clearing for corporate bonds then implies:

$$f'^{-1}(r_1) + R^h / r_1 = B_1. \tag{9}$$

From the hypothesised properties of $f(.)$, equation (9) uniquely defines the equilibrium interest rate for all positive values of B_1. The implied interest rate function, $r_1(B_1)$, is continuous and such that $r_1'(B_1) < 0$, $r_1(0) = \infty$ and $r_1(\infty) = 0$. Equations (8)–(9) then fully characterise the intermediated equilibrium price vector at date 1, (P_1, r_1), *conditional* on the amount of aggregate lending, B_1.

Note from (6) and (9) that at the point $B_1 = B_1^F$ the intermediated interest rate, $r_1(B_1)$, is greater than its fundamental analogue, τ. This can be explained as follows. For a given value of B_1, the expected asset payoff that accrues to investors in the intermediated equilibrium, R^h, is higher than the expected payoff to lenders in the fundamental equilibrium, πR^h. In consequence, risky assets are bid up in the intermediated equilibrium and safe asset investment, X_{S1}, is crowded out, which in turn raises the equilibrium interest rate, r_1 (relative to the fundamental rate, τ). The intermediated equilibrium is thus characterised by *risk shifting,* in the sense that portfolio delegation to debt-financed investors leads to an excessive share of risky asset investment, and too little safe asset investment, relative to the efficient portfolio (i.e., the fundamental equilibrium). The implications of this distortion for equilibrium asset prices and savings are further analysed in Section 3.4.

3.2 Expected return on loans

Given lenders' utility function, individual lending decisions at date 1 depend on the expected return on the loans they make to investors, denoted by ρ_1, as compared to the certain return they receive from storing, τ. Note that in general ρ_1 differs from the contracted loan rate, $r_1^l = r_1$, because of the possibility that investors will default on loans at date 2.

When investors do not default on loans (i.e., when $R = R^h$), the contracted loan rate applies and they repay lenders $r_1 B_1$. When they do default, lenders collect the residual value of the investors' portfolio, i.e., the capitalised value of corporate bonds, $r_1 X_{S1} = r_1 (B_1 - P_1)$. The *ex ante* unit loan return is thus $\pi r_1 + (1-\pi) r_1 (1 - P_1/B_1)$ or, using (8) and the interest rate function $r_1(B_1)$ defined by (9),

$$\rho_1(B_1) = r_1(B_1) - \frac{(1-\pi) R^h}{B_1} \ (>0). \tag{10}$$

Note from equations (5), (9) and (10) that the probability that investors become bankrupt at date 2, $1-\pi$, indexes the gap between the contracted and actual *ex ante* returns on savings, r_1 and ρ_1. When $\pi = 1$ the risk-shifting problem disappears since portfolio investors never default; the intermediated loan return, $\rho_1(B_1)$, is then identical to the contracted loan rate, $r_1(B_1)$, which in turn equals the fundamental interest rate, τ. When $\pi < 1$, investors' and lenders' incentives become misaligned, and a gap $(1-\pi) R^h/B_1 > 0$ appears between r_1 and ρ_1. Thus, $1-\pi$ measures both the severity of the risk-shifting problem in the economy (i.e., the extent to which investors take more risk than if they were playing with their own funds) and the implied distortion in the intermediated return on loans (i.e., $r_1 - \rho_1$).

The first term of the right-hand side of (10), $r_1(B_1)$, is the (decreasing) interest rate function defined by equation (9): an increase in B_1 raises the amount invested in the safe asset, X_{S1}, which reduces the equilibrium interest rate, $r_1 = f'(X_{S1})$, and thus the average return on loans; this is the usual 'marginal productivity effect' of aggregate savings on the loan return. In contrast, the second term, $-(1-\pi) R^h/B_1$, *increases* with B_1; this latter effect reflects the impact of the total loan amount on the average riskiness of loans as the *composition* of the optimal portfolio varies with B_1. To analyse this second effect in more detail, first use (9) to write the relationship between safe asset investment, X_{S1}, and aggregate lending, B_1, as follows:

$$B_1 = X_{S1} + R^h / f'(X_{S1}). \tag{11}$$

From (11) and assumption (1) regarding the concavity of $f(.)$, it is easy to check that an increase in B_1 raises *both* the quantity of safe assets, X_{S1}, and the share of safe asset investment in investors' portfolio, X_{S1}/B_1 (i.e., it lowers $B_1/X_{S1} = 1 + R^h/X_{S1}f'(X_{S1})$). In other words, even though an increase in B_1 lowers r_1 and thus raises asset prices, R^h/r_1, the relative size of risky asset investment, $P_1/B_1 = 1 - X_{S1}/B_1$, *decreases* as B_1 increases. This 'portfolio composition effect' in turn limits the loss to lenders in the case of investors' default and raises the *ex ante* return on loans.

Given these two effects, the crucial question is: Are there intervals of B_1 over which $\rho_1(B_1)$ may be increasing, i.e., where the portfolio composition effect dominates the marginal productivity effect? To obtain some insight into the conditions under which this is the case, solve (9) for R^h and substitute the resulting expression into (10) to obtain:

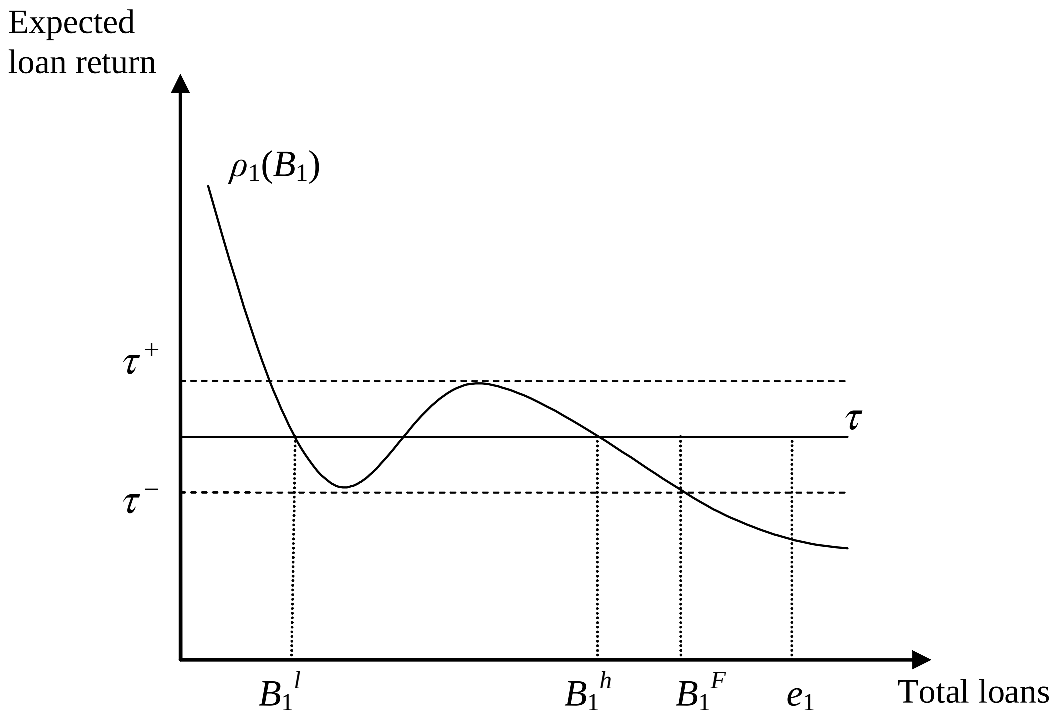
$$\rho_1(B_1) = r_1(B_1)(\pi + (1-\pi)(X_{S1}/B_1)). \tag{12}$$

Both effects are made explicit in (12). Intuitively, for the increase in X_{S1}/B_1 to dominate the decrease in $r_1(B_1)$ induced by a marginal increase in B_1, $1-\pi$ must be sufficiently large (i.e., the risk-shifting problem must be large enough), and $-r_1'(B_1)\,(>0)$ must be not too large (i.e., the marginal productivity effect must be weak enough). When this is the case, 'strategic complementarities' (in the sense of Cooper and John, 1988) in lending decisions appear, as a symmetric decision by other lenders to increase their loans to investors leads any individual lender to do the same. Proposition 1 formally establishes the conditions for such complementarities to occur in the general case, as well as for a more specific class of production functions.

Proposition 1 (Strategic complementarities). The loan return curve, $\rho_1(B_1)$, which satisfies $\rho_1(0) = \infty$ and $\rho_1(\infty) = 0$, is non-monotonic in total loans, B_1, provided π and $-f''(x)$ are not too large. In the isoelastic case where $f(x) = x^{1-\eta}/(1-\eta)$, $\eta \in (0,1)$, $\rho_1(B_1)$ has exactly one (zero) increasing interval if $2\eta + \sqrt{\pi} < (\geq)\, 1$.

For a general function $f(.)$, there may be several intervals of B_1 over which $\rho_1(B_1)$ is increasing, i.e., over which the implied $-f''(X_{S1})$ is sufficiently small (provided π is not too large). In the isoelastic case, a high value of η increases the curvature of $f(.)$ and strengthens the marginal productivity effect; thus, neither π nor η must be too large for the portfolio composition effect to dominate the marginal productivity effect. In the remainder of the

Figure 2: Loan market equilibrium



paper, we shall focus on a particularly simple case of non-monotonicity by assuming that $\rho_1(B_1)$ has one single increasing interval, as depicted in Figure 2, and as implied by the isoelastic case when $2\eta + \sqrt{\pi} < 1$ (all of our results generalise straightforwardly to the case of multiple increasing intervals).

3.3 Loan market equilibrium

Having characterised the *ex ante* loan return, ρ_1, as a function of the amount of aggregate loans, B_1, we may now analyse the way the latter is determined in equilibrium. At date 1, lenders choose the individual level of loans, $\hat{B}_1$, and individual storage, $\hat{S}_1$, to maximise expected terminal consumption, taking $\rho_1 = \rho_1(B_1)$ as given. Given the lenders' objective, they find it worthwhile increasing (decreasing) their loans to investors whenever $\rho_1 > (<)\, \tau$. Any interior equilibrium must thus satisfy $\rho_1 = \tau$. We focus on symmetric Nash equilibria, where the lending and storage plans are identical across lenders (i.e., $\hat{B}_1 = B_1$) and no lender finds it worthwhile to individually alter his own plan. The following proposition naturally follows.

Proposition 2 (Multiple equilibria). Assume that $\rho_1(B_1)$ has one increasing interval. Then there exist $\tau^- > 0$ and $\tau^+ > \tau^-$ such that if $\tau \in (0, \tau^-] \cup [\tau^+, \infty)$ then the model has a unique stable, interior equilibrium, while if $\tau \in (\tau^-, \tau^+)$ then the model has two stable, interior equilibria $B_1^l \in (0, e_1)$ and $B_1^h \in \left(B_1^l, e_1\right)$.

In short, Proposition 2 states that, given a non-monotonic loan return curve, multiplicity occurs when the return on storage takes intermediate values, while uniqueness prevails when this return is either sufficiently high (in which case only low lending is possible) or sufficiently low (in which case only high lending results). Figure 2 displays the case where $\tau \in (\tau^-, \tau^+)$, i.e., where the τ-line intersects the $\rho_1(B_1)$-curve more than once.

Recall from equation (11) that an increase in B_1 lowers marginal productivity but also reduces the share of risky assets in investors' portfolios. The low-lending equilibrium is thus characterised by a higher interest rate r_1 but also a greater share of risky assets in the portfolio, while the high-lending equilibrium is characterised by a lower interest rate but a safer average portfolio. Finally, notice that even though both equilibria yield the same *ex ante* return on loans, τ, they are always associated with different levels of interest rates, asset prices, productive investment, and (expected) date 2 output: equation (9) and the fact that $B_1^h > B_1^l$ implies that $r_1(B^h) < r_1(B^l)$. Then, denoting the asset's price by P_1^j and productive investment by X_{S1}^j when total lending is B_1^j, we have:

$$P_1^h = R^h / r_1(B^h) > P_1^l = R^h / r_1(B^l),$$
$$X_{S1}^h = f'^{-1}\left(r_1(B_1^h)\right) > X_{S1}^l = f'^{-1}\left(r_1(B_1^l)\right),$$

In short, the selection of the low-lending equilibrium raises the interest rate and depresses asset prices and productive investment, relative to the equilibrium with high lending. (More generally, there may be more than two stable equilibria if $\rho_1(B_1)$ has more than one increasing interval, but their properties are similar to the 2-equilibrium case, i.e., the higher is B_1, the lower is $r_1(B_1)$, and the higher are P_1, X_{S1} and $E_1(Y)$). Finally, note that in the high-lending equilibrium the aggregate endowment is more invested in risky assets than in the low lending equilibrium (i.e., the ratio of risky asset to safe assets, $P_1^j/(e_1 - P_1^j)$, is higher when $j = h$ than when $j = h$.)

3.4 Comparison with the fundamental equilibrium

We emphasised above that the risk-shifting problem arising under market segmentation leads investors to overinvest in risky assets, relative to the fundamental equilibrium. Proposition 3 summarises the implications of this distortion for the price of the risky asset and the amount of aggregate saving and productive investment in equilibrium.

Proposition 3 (Asset bubbles and crowding out). In both intermediated equilibria, asset prices are higher than in the fundamental equilibrium (i.e., $P_1^h > P_1^l > P_1^F$), while aggregate lending and productive investment are lower than their fundamental analogues (i.e., $B_1^l < B_1^h < B_1^F$ and $X_{S1}^l < X_{S1}^h < X_{S1}^F$).

That $P_1^j > P_1^F$, $j = l, h$, indicates that assets are overpriced at date 1 in both intermediated equilibria, i.e., both equilibria are associated with a positive *bubble* in asset prices (the bubble being larger, the larger is aggregate credit). Because investors are protected against a bad value of the asset payoff by the use of simple debt contracts, they bid up the asset and consequently raise its price and its share in equilibrium portfolios (relative to the fundamental equilibrium).

The reason why savings are lower in both intermediated equilibria than in the fundamental equilibrium (i.e., $B_1^l < B_1^h < B_1^F$) follows naturally: excessive risky-asset investment by portfolio investors implies that at $B_1 = B_1^F$ the intermediated *ex ante* loan return, $\rho_1(B_1)$, is *lower* than the fundamental return, τ. Lenders thus optimally raise storage in the intermediated equilibrium (relative to the fundamental equilibrium) up to the point where the intermediated and the fundamental returns are equal. Note that, as a consequence, a double crowding out effect is in fact at work on X_{S1} in the intermediated equilibrium. First, at $B_1 = B_1^F$ bubbly asset prices crowd out safe asset investment, X_{S1}, which raises the equilibrium interest rate, $r_1 = f'(X_{S1})$. Second, lenders' optimal reaction to the resulting price distortion is to reduce B_1 below B_1^F, which lowers X_{S1} (and raises r_1) even further.

3.5 Comparative statics and threshold effects

Our analysis thus far has focused on the existence conditions and properties of multiple equilibria. Proposition 4 below summarises how the deep parameters of the model affect the loan return curve and, by implication, which equilibrium(a) may be expected to prevail.

Proposition 4 (Effect of fundamental risk). An increase in fundamental risk, in the form of either a higher default probability (i.e., an increase in $1-\pi$ holding R^h fixed) or a higher mean preserving spread in the risky asset's payoff (i.e., a higher value of $1-\pi$ holding πR^h fixed), lowers the whole loan return curve, $\rho_1(B_1)$.

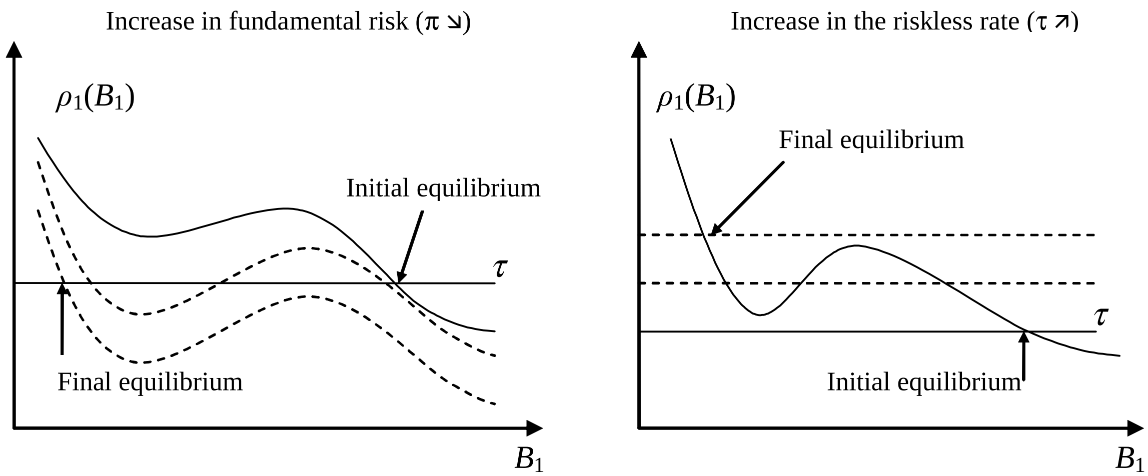
Proposition 4 summarises how changes in aggregate risk shape the loan return curve and affects the existence of the lending equilibria depicted in Figure 2. More specifically, for any given value of τ, the low-lending equilibrium B_1^l is all the more likely to exist, either jointly with the high-lending equilibrium B_1^h or as a unique equilibrium, as fundamental risk –as defined in Proposition 4– rises; conversely, the high lending equilibrium is all the more likely to exist (either in isolation or jointly with the low-lending equilibrium) as fundamental risk falls. Note that what matters here is not the location of the $\rho_1(B_1)$-curve per se but its location relative to that of the τ-line. Similar statements can thus be made about changes in τ, holding the $\rho_1(B_1)$-curve fixed: the high- (low-) lending equilibrium is all the more likely to exist when τ is low (high).

Although a proper analysis of booms and busts cycles would require a fully dynamic extension of the model, it is nevertheless instructive to explore some implications of the comparative statics properties just derived in an economy where the two-period sequence analysed so far were to repeat itself over time.[15] Imagine, for example, a situation where fundamental risk is initially low, and the implied $\rho_1(B_1)$-curve sufficiently high, to ensure the prevalence of a unique equilibrium with high lending –see the solid line in the left panel of Figure 3. Now suppose that fundamental risk (i.e., $1-\pi$) starts increasing, causing the $\rho_1(B_1)$-curve to shift downwards. At some point, a second, low-lending equilibrium appears and the initial equilibrium becomes exposed to lenders' panic, even though it may still prevail for some time if no drastic change of expectations occurs (the upper dotted line). If fundamentals continue to worsen, however, the high equilibrium vanishes and a sudden, discontinuous equilibrium change from high to low lending –a credit and asset market crash– is bound to occur (the lower dotted line). A similar jump may occur through a gradual increase in the storage rate τ, holding fundamental risk constant –see the left panel of Figure 3. If τ is sufficiently low, only high lending is possible; as τ increases, a separate, low-lending equilibrium appears, and only the low equilibrium will finally exit as τ continues

[15] see Gennotte and Leland (1990) for a similar approach.

to rise.

Figure 3: Threshold effects



We find these crash scenarios helpful in interpreting the sudden credit and asset price collapse associated with the subprime mortgage crisis that hit worldwide financial markets in August 2007. The years preceding the crisis were times of historically low interest rates, fostered by high world savings (notably from China and oil-exporting countries) and a particularly accommodative monetary policy from the Federal Reserve over most of the period. At the same time, low global inflation and sustained GDP growth, both in the US and across the world, reduced macroeconomic uncertainty and thus the perceived risk associated with holding large classes of assets –including residential property and the securitised loans that had financed their purchase. As we have just argued, both factors are conducive to a lending boom fuelled by limited default risk (that is, a high $\rho(B_1)$-curve) and low world riskless rates (i.e., a low τ-line).

The Federal Reserve initiated a round of policy tightening in 2004 that lasted until two years later, at about the time when the fundamental risk associated with subprime mortgage-based securities started to deteriorate (see Demyanyk and Van Hemert, 2008). While market participants took some time before fully realising the extent of the increased default risk, the market became aware of it at the latest in early July 2007 (Greenlaw *et al.*, 2008). In our model, the worsening of perceived risk conditions and the higher money market rate translate into a downward shift in the $\rho_1(B_1)$-curve and an upward shift in the τ-line, both of which, as we have argued, are likely to lead to financial fragility. The actual crash – our discontinuous change of equilibrium– occurred one month later, either because multiple equilibria made it possible for expectations to suddenly change in a self-fulfilling fashion, or because fundamental risk had increased so much as to make the high-lending equilibrium unsustainable.

4 Self-fulfilling financial crises

The previous section has shown that the risk shifting problem that arises under market intermediation may lead, under endogenous lending, to the existence of multiple equilibria associated with different levels of aggregate lending, interest rates, and asset prices. We now expand the time span of the model to demonstrate the possibility of a *self-fulfilling financial crisis* associated with the selection of the low-lending equilibrium at date 1 (Section 4.1). Besides offering a stochastic version of the multiple equilibria model, the self-fulfilling crisis model has two important implications. First, it generates endogenous bankruptcies in equilibrium, as the selection of low-lending/low-asset price equilibrium at the intermediate date causes the assets of initially levered investors to fall short of their liabilities (Section 4.2). Second, it uncovers some of the negative welfare consequences of crises working through the wealth effects of the crash on lenders' consumption (Section 4.3).

4.1 The three-date model

The model has now three date, 0, 1 and 2. Lenders live for 3 periods, maximise terminal consumption, and receive the endowment $e_0 > 0$ at date 0 (in addition to receiving e_1 at date 1). They face overlapping generations of two-period lived investors and entrepreneurs entering the economy at dates 0 and 1. In the following, we shall refer to 'date t investors (entrepreneurs)' as the investors (entrepreneurs) who enter the economy at date t, $t = 0, 1$, and leave it at date $t+1$. The risky asset is now assumed to be three-period lived – it is sold by the one-period lived initial asset holders at date 0 and delivers its final payoff at date 2. The production lag is of one period as before, with X_{St} units of productive investment at date t, $t = 0, 1$, yielding $f(X_{St})$ units of good at date $t+1$. Finally, we assume for simplicity that the storage technology is only available from date 1 to date 2.[16] These assumptions are meant to ensure that the intermediate date of the three-date model exhibits exactly the same equilibrium levels of lending as the initial date of the two-period model; we can then straightforwardly work backwards the equilibrium at date 0, given the possible outcomes at date 1 and the likelihood that they occur.

Crisis equilibria are constructed by randomising over the two possible lending equilibria that may prevail at date 1. More specifically, assume that, from the point of view of date 0, high lending is selected with probability $p \in (0, 1)$ at date 1, so that the 'sunspot' on which agents coordinate their expectations causes lending and asset prices to drop down to low levels with probability $1 - p$. It is assumed that at date 0 all agents share the same prior about $1 - p$, and that the latter is consistent the true probability that the crisis signal will occur at date 1 (the three-date model thus potentially has a *continuum* of stochastic equilibria indexed by the *ex ante* probability of a market crash, $1-p$). Since the asset's price at date 1 is the asset payoff accruing to date 0 investors, this uncertainty about asset prices creates a risk-shifting problem at date 0 similar to that created at date 1 by the intrinsic uncertainty about the asset's terminal payoff. This causes the asset to be bid up at date 0, with the possibility that a self-fulfilling crisis (i.e., a drop in asset prices forcing date 0 investors into bankruptcy) occurs if the low lending equilibrium is selected.

4.2 Date 0 equilibrium

Contracted loan rate. Denote by (P_0, r_0) the equilibrium asset price vector, r_0^l the contracted loan rate, and (X_{S0}, X_{R0}) the portfolio of date 0 investors. Date 0 entrepreneurs receive $f(X_{S0})$ units of goods at date 1 from investing X_{S0} in the production technology at date 0, so their optimal investment choice is such that $r_0 = f'(X_{S0})$. On the other hand, the limited liability of date 0 investors and the portfolio constraint $B_0 = X_{S0} + P_0 X_{S0}$ imply

[16] Our results can be generalised to the situation where storage is also available from date 0 to date 1, but the full analysis of this case requires substantial algebra without significantly altering our results. Under this generalisation, if the self-fulfilling uncertainty that plagues asset prices at date 1 is sufficiently strong, then it may generate multiple equilibria at date 0 –in the same way as strong fundamental uncertainty at date 2 may generate multiple equilibria at date 1. Assuming that storage is not available at date 0 amounts to ruling out this additional source of equilibrium multiplicity.

their terminal consumption (i.e., at date 1) is:

$$\max\left[r_0 X_{S0} + P_1 X_{R0} - r_0^l B_0, 0\right] = \max\left[X_{R0}\left(P_1 - r_0 P_0\right) + B_0\left(r_0 - r_0^l\right), 0\right],$$

where, given our assumption about exogenous uncertainty, P_1 is a random variable at date 0, taking on the value P_1^h with probability p (i.e., B_1^h is selected), and P_1^l otherwise (B_1^l is selected). The loan rate r_0^l must be equal to the rate on corporate bonds r_0: were r_0^l to be lower (higher) than r_0, then investors would want to borrow infinitely many (zero) units of goods to buy bonds, while the loan supply at date 0 is exactly e_0 (the expected return on loans at date 0 is non-negative, because the liquidation value of date-0 portfolios cannot be negative). Thus, any equilibrium must satisfy $r_0^l = r_0 = f\left(X_{S0}\right)$ and $B_0 = e_0$.

Asset prices and interest rate. In the equilibria that we are considering, date 0 investors default on loans when the asset price at date 1 is P_1^l, but not when it is P_1^h. Since $B_0\left(r_0 - r_0^l\right) = 0$, their terminal consumption is $X_{R0}\left(P_1^h - r_0 P_0\right) \geq 0$ with probability p and 0 otherwise. Date 0 investors choose the level of X_{R0} that maximises expected consumption, $pX_{R0}\left(P_1^h - r_0 P_0\right)$, while any potential solution to their decision problem must be such that they do not default on loans if the asset price at date 1 is P_1^h, but do default if it is P_1^l, i.e.,

$$P_1^h - r_0 P_0 \geq 0,\ P_1^l - r_0 P_0 < 0. \tag{13}$$

The demand for risky assets by date 0 investors, X_{R0}, is infinite (zero) if $P_1^h - r_0 P_0 > 0\,(< 0)$. Market clearing thus requires that the equilibrium price of the risky asset be:

$$P_0 = P_1^h / r_0, \tag{14}$$

which satisfies both inequalities in (13). Again, the interpretation of this equilibrium price is straightforward. Perfect competition for the risky asset by date 0 investors implies an asset price such that they make zero expected profit. Because they make zero profit from holding risky assets when the asset payoff is P_1^l (i.e., when they default), they must also earn zero when it is P_1^h; this is exactly what the equilibrium price P_1^h/r_0 ensures.

Aggregate lending from date 0 to date 1 is e_0. In equilibrium we have $X_{R0} = 1$ and $r_0 = f'\left(X_{S0}\right) = f'\left(e_0 - P_0\right)$. Thus, r_0 is uniquely determined by the following equation:

$$f'^{-1}\left(r_0\right) + P_1^h / r_0 = e_0, \tag{15}$$

where $P_1^h = R^h/r_1(B_1^h)$ is independent of e_0, due to the interiority of B_1^h following from assumption (2). Note from (14)-(15) that the equilibrium price vector at date 0, (P_0, r_0), is uniquely determined and does not depend on the probability of a crisis, $1-p$: as date 0 investors are protected against a bad shock to the value of their portfolio by the use of simple debt contracts, they simply disregard the lower end of the payoff distribution (i.e., the payoff P_1^l with probability $1-p$) when selecting their optimal portfolio.

4.3 Wealth and welfare effects of financial crises

Having shown the existence of a continuum of stochastic equilibria indexed by the probability of a self-fulfilling crisis, we are now in a position to study the welfare properties of these equilibria in more details. We first analyse the way in which crises affect lenders' wealth and terminal consumption, and then turn to the effect of crises on other agents' utility.

To see why lenders' wealth at date 1 is contingent on whether a crisis occurs at date 1 or not, we consider how it is affected by the possible default of date-0 investors. When these investors do not default, they owe lenders the capitalised value of outstanding debt at date 1, $r_0 e_0$. As lenders receive an endowment e_1 at date 1, their date 1 wealth if no crisis occurs is simply $W_1^h = e_1 + r_0 e_0$. When investors do default, on the contrary, lenders' wealth at date 1 is their date 1 endowment, e_1, *plus* the residual value of the date 0 investors' portfolio, $r_0 X_{0S} + P_1^l$. Using (15), lenders' date 1 wealth, W_1^j, conditional on whether a crisis occurs ($j = l$) or not ($j = h$), is thus given by:

$$W_1^j = e_1 + r_0 X_{S0} + P_1^j,\ j = l, h. \tag{16}$$

Obviously, the total quantity of goods available at date 1 is the same across equilibria, because initial capital investment, X_{S0}, is uniquely determined (i.e., it does not depend on p). This quantity amounts to lenders' date 1 endowment, e_1, *plus* entrepreneurs' production, $f(X_{S0})$, the latter being shared between date 0 entrepreneurs, who gather the surplus $f(X_{S0}) - r_0 X_{S0}$ in competitive equilibrium, and lenders, who receive $r_0 X_{S0}$ (recall that P_0 is such that date 0 investors consume zero whether $P_1 = P_1^l$ or P_1^h).[17]

[17] There are two equivalent ways of characterising lenders' budget sets at date 1: looking at their *wealth*, W_1^j is assigned to storage and lending, so that from (16) we have $W_1^j = e_1 + r_0 X_{0S} + P_1^j = S_1^j + B_1^j,\ j = l, h$; the total quantity of *goods* accruing to lenders at date 1 is ultimately shared between storage, S_1^j, and date

From condition (2) and the second inequality stated in Proposition 2, we have $B_1^j < B_1^F < W_1^j$, $j = l, h$, implying that both possible levels of wealth give rise to interior solutions for consumption-savings plans at date 1 where $\rho_1(B_1^j) = \tau$. If a crisis occurs at date 1, then lenders' wealth and lending at that date are W_1^l and B_1^l, respectively, while their expected date 2 consumption, from the point of view of date 1, is $\tau\left(W_1^l - B_1^l\right) + \rho_1 B_1^l = \tau W_1^l$. Similarly, if a crisis does *not* occur at date 1, then lenders' expected date 2 consumption level is $\tau\left(W_1^h - B_1^h\right) + \rho_1 B_1^h = \tau W_1^h$. Weighting these possible outcomes with the probabilities that they actually occur, and then using (16), we find that lenders' *ex ante* utility (i.e., their expected consumption from the point of view of date 0) depends on the crisis probability, $1 - p$, as follows:

$$
\begin{aligned}
E_0\left(\tau W_1\right) &= p\tau W_1^h + (1-p)\,\tau W_1^l \\
&= \tau\left(e_1 + r_0 X_{S0} + pP_1^h + (1-p)\,P_1^l\right).
\end{aligned}
$$

$E_0\left(\tau W_1\right)$ is decreasing in $1-p$, since $P_1^h > P_1^l$ and $e_1 + r_0 X_{S0}$, P_1^l and P_1^h do not depend on p. Note that it is the selection of the low-lending equilibrium itself that triggers the crisis which lowers lenders' wealth and future consumption. Thus, the utility loss incurred by lenders when a crisis occurs is akin to a pure coordination failure in consumption/savings decisions –rather than an exogenously-assumed destruction of value associated with the early liquidation of the long asset, as is often considered in liquidity-based theories of financial crises (e.g., Diamond and Dybvig, 1983, Allen and Gale, 1998, and Chang and Velasco, 2002).

The effect of the crisis on the utility of other agents is as follows. With respect to *investors*, Sections 3.1 and 4.2 have established that both date 0 and date 1 investors consume zero in equilibrium, whatever the realisation of extrinsic (date 1) and fundamental (date 2) uncertainty. Investors' *ex ante* welfare is thus zero in all equilibria. With respect to *entrepreneurs*, the terminal consumption of date-1 entrepreneurs is $f\left(X_{S1}\right) - X_{S1} f'\left(X_{S1}\right)$, which is increasing in X_{S1}. Since $X_{S1}^h > X_{S1}^l$ (see Section 3.3), their *ex ante* welfare, from the point of view of date 0, is $p\left(f\left(X_{S1}^h\right) - X_{S1}^h f'\left(X_{S1}^h\right)\right) + (1-p)\left(f\left(X_{S1}^l\right) - X_{S1}^l f'\left(X_{S1}^l\right)\right)$, which decreases with $1-p$. Date 0 entrepreneurs consume $f\left(X_{S0}\right) - f'\left(X_{S0}\right) X_{S0}$, where

1 investment, X_{S1}^j, so that $e_1 + r_0 X_{0S} = S_1^j + X_{S1}^j$, $j = l, h$. Since $B_1^j = X_{S1}^j + P_1^j$, these two formulations are, obviously, mutually consistent.

$X_{S0} = f'^{-1}(r_0)$ does not depend on p. Finally, *initial asset holders*' consumption is just the selling price of the asset at date 0, P_0, which is independent of p. In short, neither investors nor initial asset holders or date 0 entrepreneurs are affected by the crisis probability. Lenders are, because the crisis reduces their wealth and future consumption, and (date 1) entrepreneurs are, because low lending reduces their investment and consumed surplus.

5 Robustness

Our results were derived under stark simplifying assumptions about agents' preferences and the technologies that are available to them. We now test their robustness by relaxing our baseline assumptions regarding i) the flexibility of asset supplies (Section 5.1) and ii) the risk neutrality of agents (Section 5.2).

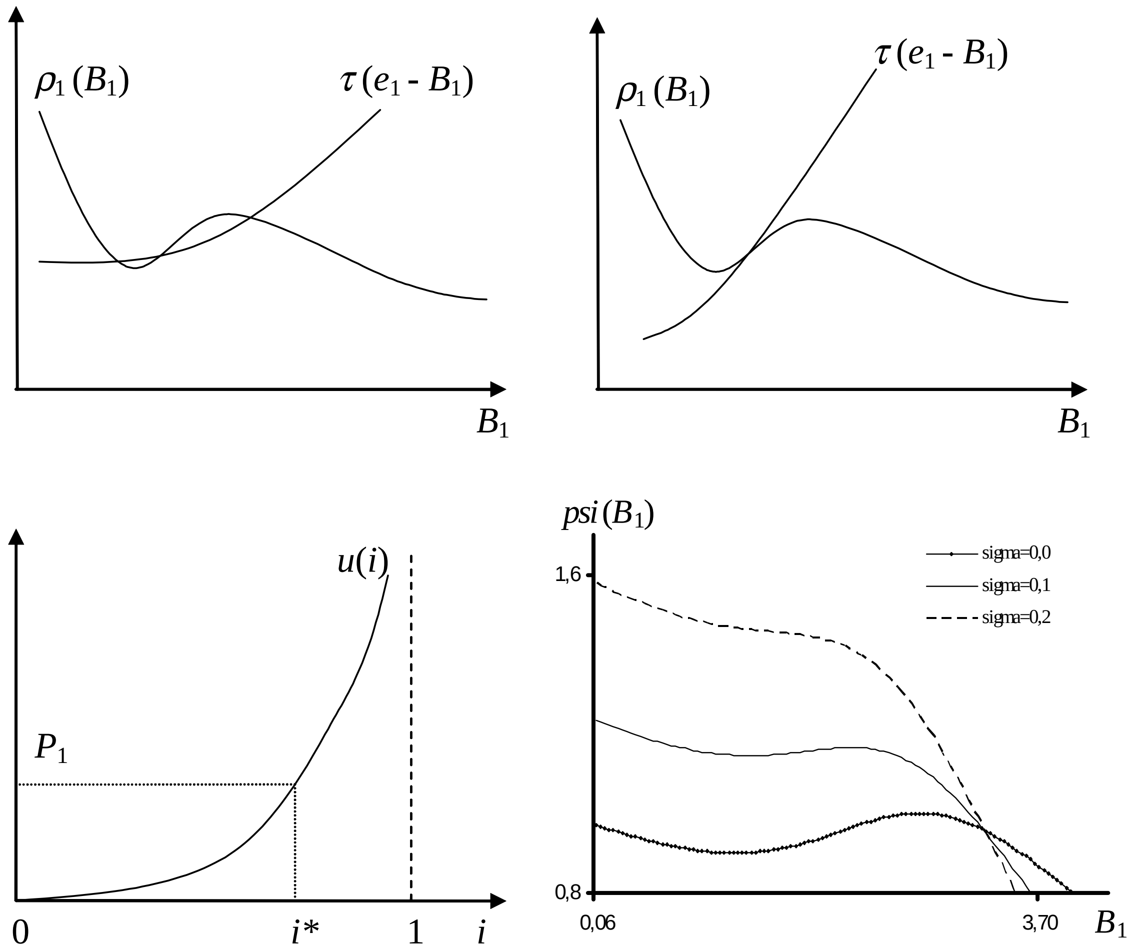
5.1 Imperfectly elastic asset supplies

Our baseline model was built on the joint assumption that risky assets were in fixed supply, while the supply of storage was completely elastic. It is thus important to gauge whether our results survive reasonable departures from these somewhat extreme assumptions.

Our model can easily accommodate a situation where the return on storage reacts to the total amount stored, i.e., where $\tau = \tau(S_1)$, $\tau'(.) < 0$ (so that $\partial\tau(e_1 - B_1)/\partial B_1 > 0$). If storage represents international liquidity, for example, this will be the case if our economy is a large, open one whose capital flows affects the world interest rate. The implied increasingness in τ with respect to B_1 is still consistent with multiple equilibria provided that the $\tau(e_1 - B_1)$-curve increases sufficiently less than the $\rho(B_1)$-curve, as is illustrated in the top left panel of Figure 4 (in contrast, in the top right panel the $\tau(e_1 - B_1)$-curve is so steep as to destroy the possibility of multiple equilibria).

Our results also continue to hold if the supply of risky assets is flexible but sufficiently less so than the production technology. To consider this possibility in the simplest possible way, suppose that initial asset holders must *produce* the risky asset at date 1 –rather that merely being endowed with it– before selling it to investors. More specifically, assume that there is a continuum of initial asset holders indexed by i and uniformly distributed along the interval $[0, 1]$. Each of them faces the binary choice of producing one unit of the risky

Figure 4: Robustness



asset or not, and the risky asset has the same payoff structure as before, i.e., $R^h > 0$ w.p. π and 0 w.p. $1 - \pi$.[18] Initial asset holders are differentiated according to the fixed cost they must incur to produce the asset, summarised by the function $u(i)$. Finally, assume that $u(.)$ is continuous over $[0, 1)$, that $u(0) = 0$, $u(1) = \infty$ and that $u'(.) > 0$ –i.e., agents are ranked in increasing order of production cost, and no two agents face exactly the same production cost (see the bottom left panel of Figure 4). Under this production technology for risky assets, asset producer i produces his asset unit if and only if $P_1 \geq u(i)$, and thus enjoys a consumption level of $\max[P_1 - u(i), 0]$. The *marginal asset producer*, denoted i^*, is

[18] We can also consider the case where the quantity of assets being produced depends on its price and where its favorable payoff, R^h, depends negatively on the total quantity of assets.produced. Here again, our results remain robust provided that this decreasing productivity effect is not too pronounced.

exactly indifferent between producing the asset or not, so that for him $P_1 = u(i^*)$. Since all producers facing production cost lower than that of the marginal producer produce exactly one asset unit, the total number of risky assets supplied is:

$$i^* = \int_0^{i^*} di = g(P_1), \tag{17}$$

where $g(.) \equiv u^{-1}(.)$, $g'(.) > 0$, and where $\xi(P_1) = P_1 g'(P_1)/g(P_1)$ is the price-elasticity of the risky asset supply. How is the equilibrium affected by this generalisation? Note first that the price equation (8) still holds, since it is determined by investors' equalisation of returns across assets. However, market clearing for corporate bonds now requires $r_1 = f'(B_1 - i^* P_1)$. Using (8) and (17), this implies:

$$f'^{-1}(r_1) + h\left(\frac{R^h}{r_1}\right) = B_1, \tag{18}$$

where $h(x) \equiv x.g(x)$. Since $h(.)$ is continuous and strictly increasing, equation (18) implicitly defines a continuous, decreasing interest rate function $r_1(B_1)$. Finally, the loan return curve $\rho_1(B_1)$ is still given by equation (10), with r_1 now defined by (18), rather than by (9). To summarise, the central difference between the endogenous asset supply specification and the baseline model is the fact that $h(x)$ is a nonlinear function in (18), whereas it was linear in equation (9). The following proposition generalises the results of Propositions 1 and 2 to the case where both storage and risky assets are in imperfectly elastic supply.

Proposition 5 (Imperfectly elastic asset supplies). For any increasing risky asset supply function $g(.)$, the loan return curve, $\rho_1(B_1)$, is non-monotonic in total loans, B_1, provided π and $-f''(x)$ are not too large. If $\rho_1(B_1)$ is increasing at least over one range of B_1, then there exists a storage return function $\tau(e_1 - B_1)$ such that multiple equilibria exist.

There is no analytical condition as simple as that stated in Proposition 1 for the isoelastic case when risky assets are in flexible supply. Nevertheless, Proposition 4 establishes the intuitive result that if $f(X_{S1})$ is sufficiently flat over some range of X_{S1}, where X_{S1} is implied by the choice of B_1 in equilibrium (see the proof of the proposition for further detail), then the marginal productivity effect may be sufficiently reduced so as to be dominated by the portfolio composition effect –even though the latter may be weaker under flexible asset supply than under fixed supply.

5.2 Risk-averse agents

The assumption of limited investor liability, coupled with the hypothesis of all agents' risk neutrality, introduces a great deal of 'risk-loving' behaviour in the economy. This naturally raises the question whether our results are still valid when agents, especially lenders, are risk-averse. To investigate this case, assume that all agents maximise a function $v(.)$ of terminal consumption, defined over $(0, \infty)$ and such that $v'(.) > 0$, $v''(.) < 0$. Entrepreneurs' choices at date 1 are not altered by this generalisation, since their terminal consumption is positive and deterministic. It is easy to check that investors' decisions are also the same as in the risk-neutral case provided that they receive an (arbitrarily small) extra terminal endowment $\tilde{e} > 0$.[19] Denoting lenders' terminal consumption by c_2, they now choose individual lending, $\hat{B}_1$, which maximises $Ev(c_2)$, taking aggregate lending, B_1, asset prices, P_1, and the interest rate, r_1, as given. If investors do not default, any individual lender having lent $\hat{B}_1$ receives the contractual repayment $r_1\hat{B}_1$ at date 2. If investors do default, this lender is entitled to a share of the residual portfolio, $r_1(B_1 - P_1)$, proportional to his share in investors' liabilities, $\hat{B}_1/B_1$. Lenders thus solve:

$$\max_{\hat{B}_1} \tau\left(e_1 - \hat{B}_1\right) + \left(\pi v(r_1\hat{B}_1) + (1-\pi)\, v\left(\hat{B}_1 \times \frac{r_1(B_1 - P_1)}{B_1}\right)\right). \tag{19}$$

Solving (19) for $\hat{B}_1$, and then using $P_1 = R^h/r_1$ and imposing symmetry across lenders ($\hat{B}_1 = B_1$), we find that any equilibrium lending level must satisfy:

$$\psi(B_1) \equiv \pi r_1 v'(r_1 B_1) + (1-\pi)\left(r_1 - \frac{R^h}{B_1}\right) v'\left(r_1 B_1 - R^h\right) = \tau, \tag{20}$$

where, from investors' optimal portfolio choice, $r_1 = r_1(B_1)$ is defined by equation (9) above. Note that when $v(c_2) = c_2$ then $\psi(B_1) = \rho_1(B_1)$ and (20) is reduced to $\rho_1(B_1) = \tau$, our equilibrium condition under risk neutrality (see Figure 2). Thus, $\psi(.)$ generalises the $\rho_1(.)$ function for the risk-averse case, and can consequently be interpreted as the 'risk-corrected' *ex ante* return that lenders expect from their loans to investors (which is τ in equilibrium).

[19] The expected utility of date 1 investors is then $(1-\pi)\, v(\tilde{e}) + \pi v\left(X_{R1}\left(R^h - r_1 P_1\right) + \tilde{e}\right)$, yielding the asset demand $\left(R^h - r_1 P_1\right) v'\left(X_{R1}\left(R^h - r_1 P_1\right) + \tilde{e}\right) = 0$; in equilibrium $X_{R1} = 1$ and $R^h - r_1 P_1 = 0$ since $v'(\tilde{e})$ is positive and finite. Similarly, the date 0 investors' asset demand is such that $\left(P_1^h - r_0 P_0\right) v'\left(X_{R0}\left(P_1^h - r_0 P_0\right) + \tilde{e}\right) = 0$, yielding (14) in equilibrium. An alternative assumption is that $\tilde{e} = 0$ but $\lim_{x \to 0} x v'(x) = 0$.

The existence of multiple equilibria requires that $\psi(.)$ be increasing over at least one interval of B_1. Since we were not able to derive any simple analytical condition ensuring that this holds, we computed the $\psi(B_1)$ function numerically for the isoelastic case, where $f(x) = x^{1-\eta}/(1-\eta)$, $\eta \in (0,1)$, and $v(c_2) = c_2^{1-\sigma}/(1-\sigma)$, $\sigma \geq 0$, for a variety of parameter values. We found that $\psi(B_1)$ may have an increasing interval if the risk-shifting problem is large enough (i.e., $1-\pi$ is not too small), and neither $f(.)$ nor $v(.)$ are too concave (i.e., neither η nor σ are too large). We know from Proposition 1 and the discussion in Section 3.2 that high values of π or η are detrimental to multiple equilibria because they make it less likely that the portfolio composition effect dominate the marginal productivity effect; a positive value of σ strengthens the marginal productivity effect further by making lenders less willing to invest in risky lending relative to the safer storage technology. For sake of illustration, the bottom right panel of Figure 4 represents the risk-corrected loan return curve when $\eta = R^h = 0.1$ and $\pi = 0.5$, for different values of σ; As σ gradually increases, the increasingness of $\psi(.)$ becomes less and less pronounced over the relevant range of B_1, until $\psi(.)$ decreases over the entire $(0,\infty)$ interval.

6 Concluding remarks

This paper offers a simple theory of self-fulfilling financial crises based on the excessive risk taking of debt-financed portfolio investors. In our model, the interplay between the amount of funds available to investors, the composition of their portfolio, and the return that they are able to offer in competitive equilibrium creates a strategic complementarity between lenders' savings decisions, which naturally gives rise to multiple equilibria associated with different levels of lending, interest rates, asset prices and future output. Expectations-driven financial crises may then occur with positive probability as soon as the economy exhibits (at least) two possible equilibrium levels of lending, and the coordination of lenders on a particular equilibrium is determined by an extraneous 'sunspot'. We showed that such crises are characterised by a self-fulfilling credit contraction, followed by a market crash, widespread failures of investors, and a fall in productive investment.

Apart from demonstrating that credit intermediation based on debt contracts is a potential source of endogenous financial instability, the model also provides new insights into the potential welfare costs of financial crises. In our model, the dramatic reduction in lending and asset prices associated with the crisis equilibrium has two implications. First, it brings about a reduction in lenders' wealth and consumption, due to a fall in the total value of their capitalised investment. Second, the credit contraction associated with the crisis causes a fall in productive investment and output, and consequently reduces entrepreneurs' profits and consumption. Thus, both savers and final producers are hurt by the financial crisis, while intermediate investors, whose risk is hedged by their limited liability, are ultimately left unharmed.

Appendix

Proof of Proposition 1

We wish to characterise the behaviour of $\rho_1(B_1)$ as total loans, B_1, vary over $(0,\infty)$. First, note that $\rho_1(B_1)$ is continuous and such that $\rho_1(\infty)=0$ and $\rho_1(0)=\infty$ (that $\rho_1(0)=\infty$ follows from Eqs. (9) and (12), which imply that $r_1(0)=\infty$ and $X_{S1}/B_1 \geq 0$). Although this indicates that $\partial\rho_1(B_1)/\partial B_1$ must be negative somewhere, the two terms on the right-hand side of (10) reveal that, over a given interval $[B_a, B_b] \subset (0,\infty)$, the change in $\rho_1(B_1)$ as a function of B_1 is of ambiguous sign.

From equation (10), we have that $\partial\rho(B_1)/\partial B_1 > 0$ if and only if

$$-r_1'(B_1)\,B_1^2 < (1-\pi)\,R^h. \tag{A1}$$

Given π and R^h, (A1) may hold if $-r_1'(B_1)$ is small enough over some interval of B_1, that is if the interest rate, r_1, is not very responsive to changes in the implied level of safe asset investment, X_{S1}. This in turn holds if $f(X_{S1})$ is 'flat enough' over the relevant range of X_{S1}, so that $r_1 = f'(X_{S1})$ responds only little to changes in X_{S1}. Using (9), together with the fact that $\partial f'^{-1}(r_1)/\partial r_1 = 1/f''(X_{S1})$, the left-hand side of (A1) yields:

$$-r_1'(B_1)\,B_1^2 = \frac{\left(R^h + X_{S1} f'(X_{S1})\right)^2}{R^h + f'(X_{S1})^2 / \left(-f''(X_{S1})\right)}\ (>0).$$

For $X_{S1} \in \left[\underline{X}, \overline{X}\right]$, i.e. when $B_1 \in \left[\underline{X} + R^h/f'(\underline{X}), \overline{X} + R^h/f'\left(\overline{X}\right)\right]$, $-r_1'(B_1)\,B_1^2$ can be made gradually smaller by decreasing the curvature of $f(.)$ over $\left[\underline{X}, \overline{X}\right]$; in this case $f'(X_{S1})$ is bounded both above and below, and $-f''(X_{S1})$ can be made arbitrarily small, producing a value of $-r_1'(B_1)\,B_1^2$ small enough for (A1) to hold (provided $\pi \neq 1$). The larger is $1-\pi$, the more likely it is that inequality (A1) is satisfied, for a given $r_1(B_1)$ function.

Consider now the isoelastic case. When $f(X_{S1}) = X_{S1}^{1-\eta}/(1-\eta)$, equation (9) becomes $B_1(r_1) = r_1^{-1/\eta} + R^h r_1^{-1}$, which in turn implies:

$$r_1'(B_1) = \frac{1}{B_1'(r_1)} = \frac{1}{(-1/\eta)\,r_1^{-1-1/\eta} - R^h r_1^{-2}},$$

where $r_1 = r_1(B_1)$. From equation (10), $\partial\rho_1(B_1)/\partial B_1 > 0\,(<0)$ when $r_1'(B_1) + (1-\pi)\,R^h/B_1^2 > 0\,(<0)$, that is, when

$$\frac{1}{(-1/\eta)\,r_1^{-1-1/\eta} - R^h r_1^{-2}} + \frac{(1-\pi)\,R^h}{(r_1^{-1/\eta} + R^h r_1^{-1})^2} > 0\,(<0)\,.$$

Defining $Y \equiv r_1^{1-1/\eta}$ and rearranging, we find that $\rho_1(B_1)$ increases (decreases) when

$$\Psi(Y) = Y^2 + R^h \left(2 - \frac{1-\pi}{\eta}\right) Y + \pi \left(R^h\right)^2 < 0\,(> 0)\,.$$

The expression $\Psi(Y)$ changes sign over $(0, \infty)$ if $\Psi(Y) = 0$ has two real roots, including at least one positive root. A necessary condition for this to hold is that the discriminant of $\Psi(Y) = 0$ be positive, i.e., the following inequality must hold:

$$1 + 4\eta(\eta - 1) > \pi. \tag{A2}$$

When (A2) holds, the roots Y_a, Y_b of $\Psi(Y) = 0$ are:

$$Y_{a,b} = \frac{R^h}{2}\left(\left(\frac{1-\pi}{\eta} - 2\right) \mp \sqrt{\left(\frac{1-\pi}{\eta} - 2\right)^2 - 4\pi}\right).$$

Both roots are positive (negative) if $1 - 2\eta > (<)\,\pi$. Combined with inequality (A2), this means that $\Psi(Y)$ changes signs over $(0, \infty)$ if and only if

$$2\eta + \sqrt{\pi} < 1. \tag{A3}$$

$\Psi(Y)$ is negative for $Y \in (Y_a, Y_b)$, and positive for $Y \in (0, Y_a) \cup (Y_b, \infty)$. Since $Y = r_1^{1-1/\eta}$, this means that $\Psi(Y)$ is negative for intermediate values of r_1 and positive otherwise. Using (9) again, this in turn implies that, provided (A3) holds, $\rho_1(B_1)$ is strictly increasing for intermediate values of B_1 and strictly decreasing otherwise. When (A3) does not hold, then $\Psi(Y)$ is non-negative and $\rho_1(B_1)$ is decreasing or flat over $(0, \infty)$.

Proof of Proposition 2

The existence and number of equilibria as a function of τ is straightforward. We focus on the interiority and stability of equilibria when $\tau \in (\tau^-, \tau^+)$, but similar arguments can be used to establish stability and interiority when uniqueness prevails. *Interiority.* We want to establish that $0 < B_1^l < B_1^h < e_1$. Since B_1^l and B_1^h can only be positive (otherwise ρ_1 would be infinite) and $B^F < e_1$ by assumption (2), a sufficient condition for interiority is that $B_1^j < B^F$, $j = l, h$. To prove that this is the case, first use the fact that $\rho_1(B_1^j) = \tau$, $j = l, h$, together with equations (9) and (10), to rewrite B_1^j as follows:

$$B_1^j = \frac{r_1(B_1^j)}{\tau} f'^{-1}\left(r_1(B_1^j)\right) + \frac{\pi R^h}{\tau},\ j = l, h.$$

Comparing the latter equation with (6), we find that $B_1^j < B_1^F$ if and only if

$$r_1(B_1^j)f'^{-1}\left(r_1(B_1^j)\right) < \tau f'^{-1}(\tau),\ j = l, h.$$

The expression $r_1 f'^{-1}(r_1)$ falls with r_1 since $f'^{-1}(r_1)+r_1 f'^{-1\prime}(r_1) = X_{S1}+f'(X_{S1})/f''(X_{S1})$ is negative by assumption (1). Thus, $r_1 f'^{-1}(r_1) < (\tau) f'^{-1}(\tau)$ if and only if $r_1(B_1^j) > \tau,\ j = l, h$, which is necessarily true from (10) and the fact that $\rho_1(B_1^j) = \tau$. *Stability.* B_1^l and B_1^h are (locally) stable since a symmetric marginal move away from equilibrium by all lenders alters the loan return in such a way as to move the economy back to equilibrium: with $\epsilon > 0$ arbitrarily small, $\rho(B_1^j+\epsilon) < \tau$ and $\rho(B_1^j-\epsilon) > \tau,\ j = l, h$. In contrast, the value of B_1 where the $\rho_1(B_1)$-curve crosses the τ-line from below, say $\tilde{B}_1$, is not stable since $\rho(\tilde{B}_1+\epsilon) > \tau$ and $\rho(\tilde{B}_1-\epsilon) < \tau$ ($\tilde{B}_1$ is still a Nash equilibrium, however, since at this point $\rho_1 = \tau$, making a unilateral deviation from $\hat{B}_1 = \tilde{B}_1$ unprofitable). Notice that in the knife-edge cases where $\tau = \tau^-$ or $\tau = \tau^+$ the model has three equilibria, of which only one is stable.

Proof of Proposition 3

Comparing equations (4) and (8), we have that $P_1^j > P_1^F$, $j = l, h$, if and only if

$$\pi r_1(B_1^j) < \tau,\ j = l, h.$$

In equilibrium, $\rho_1(B_1^j) = \tau$. Then, substituting (12) into the above inequality, we find that $P_1^j > P_1^F$ if and only if $X_{S1}^j/B_1^j > 0$, which is always true whether $j = l$ or h. The second inequality is established in the proof of Proposition 2. There it is also showed that $r_1(B_1^j) > \tau$, implying that $X_{S1}^j = f'^{-1}(r_1(B_1^j)) < X_{S1}^F = f'^{-1}(\tau)$, $j = l,h$.

Proof of Proposition 4

To compute effects of changes in π and R^h on $\rho_1(B_1)$, totally differentiate (9) and (10) at any given level of lending B_1 (so that $dB_1 = 0$), to find:

$$\left(\frac{\partial f'^{-1}(r_1)}{\partial r_1} - \frac{R^h}{r_1^2}\right) dr_1 + \left(\frac{1}{r_1}\right) dR^h = 0, \tag{A4}$$

$$d\rho_1 = \left(\left.\frac{\partial r_1}{\partial R^h}\right|_{B_1} - \frac{1-\pi}{B_1}\right) dR^h + \left(\frac{R^h}{B_1}\right) d\pi. \tag{A5}$$

Keeping R^h fixed, equation (A5) gives:

$$\left.\frac{\partial \rho_1(B_1)}{\partial \pi}\right|_{R^h,B_1} = \left(\frac{R^h}{B_1}\right) d\pi > 0.$$

Let us now turn to the case of a mean-preserving increase in fundamental risk (i.e., πR^h, rather than R^h, is held fixed). Since $\partial f'^{-1}(r_1)/\partial r_1 = 1/f''(X_{S1})$, equation (A4) gives:

$$\left.\frac{\partial r_1}{\partial R^h}\right|_{B_1} = \frac{f'(X_{S1})}{R^h + f'(X_{S1})^2/(-f''(X_{S1}))}.$$

Substituting this and the mean preserving condition $\pi dR^h = -R^h d\pi$ into (A5), we find:

$$d\rho_1 = \frac{R^h}{\pi}\left(\frac{1}{B_1} - \frac{f'(X_{S1})}{R^h + f'(X_{S1})^2/(-f''(X_{S1}))}\right) d\pi.$$

Then, using assumption (1), equation (9) again and rearranging, we obtain:

$$\left.\frac{\partial \rho_1(B_1)}{\partial \pi}\right|_{\pi R^h,B_1} = \frac{R^h f'(X_{S1})^2 (1-\eta(X_{S1}))}{\pi B_1 \left(f'(X_{S1})^2 - R^h f''(X_{S1})\right)} > 0.$$

Thus, whether R^h or πR^h are held constant, an increase in $1-\pi$ lowers the $\rho_1(B_1)$-curve.

Proof of Proposition 5

This is just a generalisation of the proofs of Propositions 1 and 2. The $\rho(B_1)$-curve is increasing if and only if $-r_1'(B_1)B_1^2 < (1-\pi)R^h$, where $r_1(B_1)$ is implicitly defined by (18). The inequality thus becomes:

$$-r_1'(B_1)B_1^2 = \frac{B_1^2}{B_1'(r_1)} = \frac{\left(X_{S1}f'(X_{S1}) + R^h g\left(\frac{R^h}{f'(X_{S1})}\right)\right)^2}{R^h h'\left(\frac{R^h}{f'(X_{S1})}\right) - \frac{f'(X_{S1})^2}{f''(X_{S1})}} < (1-\pi)R^h$$

Take any range of X_{S1}, $\left[\underline{X}, \overline{X}\right]$. Over this interval, decreasing the curvature of $f(X_{S1})$ reduces the variability of $f'(X_{S1})$ (and renders it a constant in the limit) and increases the ratio $-f'(X_{S1})^2/f''(X_{S1})$ (to infinity in the limit), thereby producing a fall in $-r_1'(B_1)B_1^2$ (to zero in the limit) for *any* increasing function $g(.)$. Now using the fact that $r_1 = f'(X_{S1})$ we may rewrite the bond-market equilibrium as follows:

$$B_1(X_{S1}) = X_{S1} + h\left(\frac{R^h}{f'(X_{S1})}\right).$$

Since $f'(.)$ is decreasing in X_{S1} and $h(.)$ is increasing in $R^h/f'(X_{S1})$, X_{S1} is increasing in B_1 and thus uniquely determined by B_1. Thus, provided that $-f''(X_{S1})$ and π are sufficiently small, $\rho(B_1)$ will be increasing over the interval $\left[B_1(\underline{X}), B_1(\overline{X})\right]$. Then, if $\rho(B_1)$ has (at least) one increasing interval, there are $\tau(e_1 - B_1)$ curves in the (B_1,τ) plane that cross the $\rho(B_1)$-curve more than once.

References

Adrian, T. and Shin, H.S., 2007. Liquidity, monetary policy and financial cycles, Current Issues in Economics and Finance 14(1), 1-7.

Allen, F. and Gale, D., 2000. Bubbles and crises. Economic Journal 110, 236-55.

Allen, F. and Gale, D., 1999. Bubbles, crises, and policy. Oxford Review of Economic Policy 15, 9-18.

Allen, F. and Gale, D., 1998. Optimal financial crises. Journal of Finance 53, 1245-84.

Aghion, P., Bacchetta, P. and Banerjee, A., 2004. A corporate balance-sheet approach to currency crises. Journal of Economic Theory 119, 6-30.

Aghion, P., Bacchetta, P. and Banerjee, A., 2001. Currency crises and monetary policy in an economy with credit constraints. European Economic Review 45, 1121-1150.

Barlevy, G., 2008. A leveraged-based model of speculative bubbles. Federal Reserve Bank of Chicago Working Paper 2008-01.

Biais, B. and Casamatta, C., 1999. Optimal leverage and aggregate investment. Journal of Finance 54(4), 1291-1323.

Bordo, M.D., 2007. The crisis of 2007: The same old story, only the players have changed, unpublished manuscript.

Borio, E.V., Kennedy, N. and Prowse, S. D., 1994. Exploring aggregate asset price fluctuations across countries: Measurement, determinants and monetary policy implications. BIS Working Paper no 40, April.

Caballero, R.J. and Krishnamurthy, A., 2006. Bubbles and capital flow volatility: Causes and risk management. Journal of Monetary Economics 53, 35-53.

Calvo, G.A., 1998. Capital flows and capital-market crises: The simple economics of sudden stops. Journal of Applied Economics 1, 35-54.

Challe, E., 2004. Sunspots and predictable asset returns. Journal of Economic Theory 115, 182-190.

Chang, R. and Velasco, A., 2002. A model of financial crises in emerging markets. Quarterly Journal of Economics 116, 489-457.

Chari, V.V. and Kehoe, P.J., 2003. Hot money. Journal of Political Economy 111, 1262-92.

Chevalier, J. and Ellison, G., 1997. Risk taking by mutual funds as a response to incentives. Journal of Political Economy 105, 1167-1200.

Cooper, R. and John, A., 1988. Coordinating coordination failures in Keynesian models. Quarterly Journal of Economics 103, 441-464.

Corsetti, G., Pesenti, P. and Roubini, N., 1999. What caused the Asian currency and financial crisis? Japan and the World Economy 11(3), 305-373.

Demyanyk, Y. and Van Hemert, O., 2008. Understanding the subprime mortgage crisis, Federal Reserve Bank of St. Louis, February 29.

Diamond, D.W. and Dybvig, P.H., 1983. Bank runs, deposit insurance, and liquidity. Journal of Political Economy 91, 401-19.

Edison, H.J., Luangaram, P. and Miller, M., 2000. Asset bubbles, leverage and 'lifeboats': Elements of the East Asian crisis. Economic Journal 110, 309-334.

Gennotte, G. and Leland, H., 1990. Market liquidity, hedging, and crashes. American Economic Review 80, 999-1021.

Greenlaw, D., Hatzius, J., Kashyap, A.K. and Shin, H.S., 2008. Leveraged losses: Lessons from the mortgage market meltdown. US Monetary Policy Forum, February 29.

International Monetary Fund, 2008. Global Financial Stability Report, April.

Jensen, M.C. and Meckling, W.H., 1976. Theory of the firm: Managerial behaviour, agency costs and ownership structure. Journal of Financial Economics 3, 305-360.

Kaminsky, G.L., 1999. Currency and banking crises: The early warnings of distress. IMF Working Paper 99/178, December.

Kaminsky G.L. and Reinhart, C.M., 1999. The twin crises: The causes of banking and balance-of-payments problems. American Economic Review 89, 473-500.

Kaminsky G.L. and Reinhart, C.M., 1998. Financial crises in Asia and Latin America: Then and now. American Economic Review 88, 444-448.

Obstfeld, M., 1996. Models of currency crises with self-fulfilling features. European Economic Review 40, 1037-1047.

Palomino, F. and Prat, A., 2003. Risk taking and optimal contracts for money managers. RAND Journal of Economics 34(1), 113-137.

Summers, L.H., 2000. International financial crisis: Causes, prevention, and cures. American Economic Review 90, 1-16.

Tirole, J., 1985. Asset bubbles and overlapping generations. Econometrica 53, 1499-1528.

Velasco, A., 1996. Fixed exchange rates: Credibility, flexibility and multiplicity. European Economic Review 40, 1023-1035.