Incomplete Markets and the Output-Inflation Tradeoff

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Abstract: This paper analyses the effects of money shocks on macroeconomic aggregates in a tractable flexible-price, incomplete-markets environment that generates persistent wealth inequalities amongst agents. In this framework, current inflation redistribute wealth from the cash-rich employed to the cash-poor unemployed and induce the former to increase their labour supply in order to maintain their desired levels of consumption and precautionary savings. If the shocks are persistent, however, they also raise inflation expectations and thus deter the employed from saving and supplying labour. We relate the strength of these two inflation taxes to the underlying parameters of the model and study how they compete in determining the overall sign and slope of the implied 'output-inflation tradeoff' relation.

Keywords: Incomplete markets. Borrowing constraints. Short-run non-neutrality

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1 Introduction

The central feature of incomplete market economies is that the financial wealth accumulated by any agent depends on the sequence of idiosyncratic shocks that this agent has faced – whether these shocks affect labour incomes, trading opportunities, production possibilities, preferences, etc. Monetary economies having this property thus naturally generate heterogenous cash holdings in equilibrium. As is now well understood, in this context stochastic lump-sum money injections that raise the price level redistribute wealth across agents and have nonneutral effects on real allocations (see Scheinkman and Weiss, 1986, and Berentsen *et al.*, 2005). In short, under heterogenous cash balances and lump sum injections an "intratemporal inflation tax" is at work that redistributes wealth within the period and is ultimately responsible for the short-run nonneutrality of money.

By construction, monetary models belonging to the complete-markets, representative agent tradition preclude such redistributive effects. They do not imply, however, that money shocks are neutral, even with perfectly flexible prices. In such economies future inflation reduces the expected return on accumulated cash balances and deters savings and labour supply (see, for example, Cooley and Hansen, 1989, on the cash-in-advance model; and Walsh, 2003: 2, on the money-in-the-utility model). Note that this nonneutrality mechanism relies on *expected* rather than current inflation and thus requires money growth shocks to display some persistence. Hence these models can only feature an "intertemporal inflation tax", as opposed to the intratemporal tax working through contemporaneous wealth redistribution.

This paper analyses the real effects of aggregate monetary shocks within a tractable flexible-price, incomplete-markets environment where both the intratemporal and the intertemporal inflation taxes are operative and compete in determining the overall response of current output and inflation to nominal shocks –a relation commonly referred to as the "output-inflation tradeoff", after Lucas (1973) and Ball *et al.* (1988).[1] More formally, our framework allows us to disentangle three potential effects of monetary shocks on inflation and aggregates. First, we have an intratemporal inflation tax, because monetary injections redistribute wealth from relatively cash-rich households, who pay the tax, towards relatively poorer households, who enjoy the corresponding subsidy; this induces the former to

[1] Note that this tradeoff differs from the traditional notion of "Phillips curve", which relates changes in nominal wage growth or inflation to changes in unemployment. While our model does generate positive unemployment, the latter remains constant over time (i.e., individual labour supply, rather than labour market participation, is time-varying.)

replete their wealth by raising labour supply, which ultimately raises current output. The intratemporal tax thus induces a positive output-inflation relation, but for reasons that differ sharply from traditional theories such as price stickiness (e.g., Ball *et al.*, 1988) or imperfect information (Lucas, 1973).

Second, the intertemporal tax is also at work, because persistence in the stochastic process for money growth raises expected inflation and discourages asset accumulation and work, once a shock has occurred; this effect induces *negative*, rather than positive, hours and output responses to money growth shocks. Hence the actual slope of the output-inflation tradeoff is ultimately determined by the relative strengths of the intratemporal and intertemporal inflation taxes, which notably depend on mean inflation and the persistence of shocks (through their effects on the cross-sectional distribution of wealth.) In particular, sufficiently high levels of mean inflation and/or persistence may lead to a *reversal* of the slope of the tradeoff; and even this reversal does not occur, their mitigating effect may severely limit the economic stimulus induced by current inflation.

Third, the two taxes are in general not independent, for the impact of the shock in general depends on expectations about how taxes and subsidies will be distributed across agents *in the future*. We isolate the interactions between the two taxes by considering asymmetric injections, whereby a particular class of agents receives more newly issued money than does the average agent. Consider, for example, the case where money injections are both persistent and baised towards relatively poorer households. On the one hand, this raises the intratemporal tax paid by the rich, which reinforces the output response to the shock. On the other hand, this provides income insurance to the richer –who may become poorer in the future– and lowers their precautionary money demand; this effect pushes them to reduce their labour supply whilst at the same time increasing the nominal price impact of the shock. Thus, while the overall impact of the bias on output is in general ambiguous, current inflation is all the more responsive to the shock that this bias is large –conversely, a bias favouring relatively richer agents raises money demand and is associated with a more gradual impact of money shocks on inflation.

We carry out our analysis within a Bewley-type monetary model, i.e., where money serves as a short-run store of value allowing agents to self-insure against idiosyncratic income fluctuations. As was first shown by Bewley (1980, 1983), this role for money arises naturally in environments where insurance markets are missing and agents cannot borrow against future income. In our model, idiosyncratic labour income risk is rooted in the transitions of individual households between 'employment', a status where they can freely choose the labour supply they rent out to competitive firms, and 'unemployment', where households receive a fixed subsidy lower than equilibrium wage income. Households' optimal response to this uninsurable income difference is to vary precautionary wealth: the employed accumulate wealth to provide for future consumption, while the unemployed dis-save to provide for current consumption. While keeping track of heterogenous variations in asset holdings stemming from the precautionary motive is intrinsically complicated and often intractable, we ensure analytical tractability by constructing a Bewley model that endogenously generates a cross-sectional distribution with a finite number of wealth states. For example, in our baseline equilibrium the wealth distribution has two states: all employed households accumulate the same quantity of real money balances (i.e., regardless of their employment history), while unemployed households all end the current period with zero wealth (i.e., the borrowing constraint forces them to liquidate their cash positions, regardless of their employment history). While we mostly focus on this class of equilibria here, we also show how this approach to limiting cross-sectional heterogeneity can be generalised to allow for more than two wealth states by allowing gradual, rather than immediate, asset liquidation by unemployed households. As far we are aware, these families of equilibria with simple cross-sectional distributions are new to the heterogenous-agents literature and may, we hope, be useful for the better understanding of the impact of other types of aggregate shocks under imperfect income insurance.

Related literature. Our framework is related to several existing monetary models with heterogenous agents. Inasmuch as we are using the Bewley framework to generate cross-sectional wealth dispersion, our analysis follows those of Bewley (1980), Scheinkman and Weiss (1986), Kehoe *et al.* (1992), Imrohoroglu (1992), and Akyol (2004). All these contributions except that of Scheinkman and Weiss (1986) focus on the optimal long-run inflation rate (i.e., optimal deviations from the "Friedman rule") and do not analyse the short-run non-neutrality of money. There are at least three differences between our work and that of Scheinkman and Weiss (1986). First, they consider a two-agent model where changes in individual employment status are perfectly synchronised (i.e., agents switch roles as the idiosyncratic shock occurs); in contrast, our model has unsynchronised changes in employment status. Second, their model generates an infinite-dimensional wealth distribution, which did not allow them to derive the output-inflation tradeoff, let alone relate its size to the underlying deep parameters of the model as we do. Third, they only allow for i.i.d. aggregate shocks, while our goal is precisely to study how expected persistence may alter the impact of these shocks. Given the lack of tractability of Bewley models with infinite-state wealth distributions, an alternative approach to the one we follow is to solve them computationally. However, computational limitations have thus far limited the applicability of this approach to the study of optimal steady-state inflation, again leaving aside the analysis of the short-run effects of inflation shocks (e.g., Imrohoroglu, 1992, and Akyol, 2004.)

The non-neutrality of inflation working through wealth redistribution has been explored in frameworks that are different from Bewley models but which also generate wealth dispersion in equilibrium. In particular, *search-theoretic* models, in which agents face idiosyncratic trading opportunities and need cash to facilitate future trades, typically generate heterogenous cash holdings –see, for example, Green and Zhou (1999), Camera and Corbae (1999), and Molico (2006). There is an interesting correspondence between these models and ours, regarding both their assumptions (e.g., uninsurable, idiosyncratic shocks) and their implications (i.e., cross-sectional heterogeneity, nonneutrality of average money growth). One key feature of the above-mentioned papers is that they have a quantity of money that grows deterministically, rather than stochastically as in our model. Moreover, and as discussed by Molico (2006), search-theoretic models generate large-dimensional heterogeneity unless strong simplifying assumptions are made that make the distribution of money holdings degenerate. In this respect, that our model allows us to *gradually* increase cross-sectional dispersion (i.e., by adding more and more wealth states) hopefully offers a good compromise between large-dimensional and degenerate cross-sectional distributions.

Related to search models are *multiple trading rounds* models –with or without matching frictions–, where idiosyncratic shocks render money holdings heterogenous in a particular subperiod, but where periodic reinsurance reinstates the homogeneity of holdings by the end of every date. While originally introduced by Lagos and Wright (2005) to measure the welfare cost of (deterministic) lump-sum money injections, it was subsequently modified by Berentsen *et al.* (2005) to allow for *stochastic* injections. Our model shares a number of features with that of Berentsen *et al.* For example, both models have agents facing uninsurable shocks and limited access to credit markets (both of which entails heterogenous cash holdings), but who are allowed to periodically replete their wealth by going through a working stage where labour supply is perfectly elatic (which drastically *limits* cross-sectional heterogeneity.) A central feature of our model, however, is that not all agents can reinsure at the end of the period, due to the persistence in idiosyncratic employment statuses. Consequently, wealth inequalities are carried over from one period to the next and we can study how it is affected by, and more generally interact with, persistent *aggregate* shocks. This is in sharp contrast to Berentsen *et al.*, where both idiosyncratic and aggregate shocks are i.i.d. and the source of nonneutrality is static in nature.

Finally, within the *overlapping-generations* tradition, Doepke and Schneider (2006) have recently looked at the aggregate effects of inter-cohort redistribution and find a negative effect of inflation shocks on output in the short and medium runs; in contrast, our model can generate a positive short-run relation between inflation and output based on intra-cohort wealth redistribution.

The remainder of the paper is organised as follows. Section 2 introduces the model and spells out its optimality and market-clearing conditions. Section 3 derives our baseline, two-wealth state equilibrium. Section 4 analyses the properties of the short-run output-inflation tradeoff generated by the model and how it is affected by mean inflation, the persistence of shocks, and the possible asymmetries in money injections. Section 5 considers equilibria with more than two wealth states, and Section 6 concludes.

2 The model

2.1 Technology, market structure and uncertainty

The economy is populated by a large number of firms, as well as a unit mass of infinitely-lived households $i \in [0,1]$, all interacting in perfectly competitive labour and goods markets. Firms produce output, y_t, from labour input, l_t, using the CRS technology $y_t = l_t$; they thus adjust labour demand up to the point where the real wage is equal to 1. There are two sources of uncertainty about household income in the model: one relating to their *employment status* and the other to the *fiat money* that they receive from the monetary authority.

Unemployment shocks. In every period each household can be either employed or unemployed. We denote by χ_t^i the status of household i at date t, where $\chi_t^i = 1$ if the household is employed and $\chi_t^i = 0$ if the household is unemployed. Households switch randomly between these two states, with $\alpha = \Pr(\chi_{t+1}^i = 1 | \chi_t^i = 1)$ and $\rho = \Pr(\chi_{t+1}^i = 0 | \chi_t^i = 0)$, $(\alpha, \rho) \in (0,1)^2$ being the probabilities of staying employed and unemployed, respectively. Given this Markov chain for individual states, the asymptotic unemployment rate is:

$$U = (1-\alpha) / (2-\alpha-\rho) . \tag{1}$$

Crucially, we limit the ability of households to diversify this idiosyncratic unemployment risk away by assuming that it is *uninsurable* and that agents *cannot borrow* against future labour income.

Money shocks. Let M_t denote the nominal money supply at the end of date t, $\tau_t = M_t/M_{t-1}$ the aggregate rate money growth, and P_t the nominal price level. Newly-issued money is injected into the economy through lump sum transfers, with household i receiving a nominal amount $P_t \gamma_t^i$ from the monetary authorities at date t. The implied *aggregate* real money injection, $\gamma_t = \int_0^1 \gamma_t^i di$, is related to money growth as follows:

$$\gamma_t = \frac{M_t - M_{t-1}}{P_t} = \frac{m_{t-1}(\tau_t - 1)}{1 + \pi_t}, \tag{2}$$

where $m_t \equiv M_t/P_t$ and $\pi_t \equiv P_t/P_{t-1} - 1$ denote real money supply and the inflation rate at date t, respectively. Our baseline assumption is that money injections are symmetric across agents (i.e., $\gamma_t^i = \gamma_t \ \forall i$); we depart from this assumption in Section 3.4, where asymmetric injections based on employment status are considered.[2] Finally, from the definition of τ_t the law of motion for real money supply is

$$m_t = m_{t-1}\tau_t / (1 + \pi_t). \tag{3}$$

2.2 Households' behaviour

The household's instantaneous utility function is $u(c) - \phi l$, where c is consumption, l is labour supply, $\phi > 0$ a scale parameter, and where u is a C^2 function satisfying $u'(c) > 0$, $u''(c) < 0$ and $\sigma(c) \equiv -cu''(c)/u'(c) \leq 1 \ \forall c \geq 0$ (i.e. consumption and leisure are not gross complements). Fiat money is the only outside asset available and, since private borrowing is prohibited, the only asset that households can use to smooth consumption. Employed households (i.e. those for whom $\chi_t^i = 1$) choose their labour supply, l_t^i, at the current wage rate ($= 1$), while unemployed households (i.e., for whom $\chi_t^i = 0$) earn no labour income but a fixed amount of 'home production', $\delta > 0$.[3] Let M_t^i denote the nominal money holdings of

[2] Our baseline assumption that tranfers are symmetric and lump sum follows much of the literature on Bewley models (e.g., Scheinkman and Weiss, 1986; Kehoe *et al.*, 1992; Imrohoroglu, 1992); it can be justified on the grounds that monetary authorities are not sufficiently well informed to treat individual agents differently, as is discussed in Kehoe *et al.* (1992) and Levine (1991). However, Section 3.4 shows that asymmetric injections generate non trivial effects, both static and dynamic, of nominal shocks.

[3] Alternatively, δ can be interpreted as an unemployment subsidy financed through a compulsory lump-sum contribution, $e = U\delta/(1-U)$, paid by all employed households and ensuring the equilibrium of the

household i at the end of date t, and $m_t^i \equiv M_t^i/P_t$ the corresponding real money holdings (by convention, we denote by M_{-1}^i the nominal money holdings of household i at the beginning of date 0). Thus, household i's problem is to choose consumption, c_t^i, labour supply, l_t^i, and real money holdings, m_t^i so as to maximise $E_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t^i\right) - \phi l_t^i \right)$, with $\beta \in (0,1)$, and subject to:

$$P_t c_t^i + M_t^i = M_{t-1}^i + P_t \left(\chi_t^i l_t^i + \left(1 - \chi_t^i\right) \delta + \gamma_t^i \right), \tag{4}$$

$$c_t^i,\ l_t^i,\ M_t^i \geq 0. \tag{5}$$

Equation (4) is the nominal budget constraint of household i at date t, while the last inequality in (5) implies that agents cannot have negative asset holdings. The Lagrangian function associated with household i's problem, formulated in real terms, is:

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t^i\right) - \phi l_t^i + \varphi_t^i m_t^i + \eta_t^i \left(\frac{m_{t-1}^i}{1+\pi_t} + \chi_t^i l_t^i + \left(1-\chi_t^i\right)\delta + \gamma_t^i - c_t^i - m_t^i \right) \right),$$

where η_t^i and φ_t^i are nonnegative Lagrange multipliers (we check below that the non-negativity constraints on c_t^i and l_t^i are always satisfied in the equilibrium under consideration). The optimality conditions for this programme are :

$$u'\left(c_t^i\right) = \eta_t^i, \tag{6}$$

$$\eta_t^i = \phi \text{ if } \chi_t^i = 1 \text{ and } l_t^i = 0 \text{ if } \chi_t^i = 0, \tag{7}$$

$$u'\left(c_t^i\right) = \beta E_t \left(\frac{u'\left(c_{t+1}^i\right)}{1+\pi_{t+1}} \right) + \varphi_t^i, \tag{8}$$

$$\varphi_t m_t^i = 0, \tag{9}$$

$$\lim_{t\to\infty} \beta^t u'\left(c_t^i\right) m_t^i = 0. \tag{10}$$

Equation (6) defines household i's marginal utility, while (7) and (8) are the intratemporal and intertemporal optimality conditions, respectively. Equation (9) states that either the borrowing constraint is binding for household i ($\varphi_t^i > 0$), implying that cash holdings are zero ($m_t^i = 0$), or the constraint is slack ($\varphi_t^i = 0$) and the household uses real balances to smooth consumption over time ($m_t^i \geq 0$). Finally, condition (10) is a transversality condition that will always hold along the equilibria that we will consider.

unemployment insurance scheme in the steady state. In this case, steady-state labour supply and output are higher than under the home-production interpretation (as working households attempt to offset the wealth effect of social-security contributions by increasing their labour income), but the dynamic behaviour of the economy faced with aggregate uncertainty is unchanged.

2.3 Market clearing

Equilibrium in the market for goods requires that, at each date, the sum of each type of agent's consumption be equal to total production, with the latter being the sum of individual labour supplies and home production. On the other hand, equilibrium in the money market simply requires that individual real holdings sum up to the aggregate real money supply. In short, market clearing requires, for all t,

$$\int_0^1 \left(\chi_t^i l_t^i + \left(1-\chi_t^i\right)\delta\right) di = \int_0^1 c_t^i di \quad \text{and} \quad m_t = \int_0^1 m_t^i di,$$

where the summation operator $\int$ is over individual households.

An *equilibrium* is defined as sequences of individual consumption levels, $\{c_t^i\}_{t=0}^{\infty}$, individual real money holdings, $\{m_t^i\}_{t=0}^{\infty}$, individual labour supplies, $\{l_t^i\}_{t=0}^{\infty}$, $i \in [0,1]$, and aggregate variables, $\{y_t, m_t, \pi_t\}_{t=0}^{\infty}$, such that the optimality conditions (6)-(10) hold for every household i, and the goods and money markets clear, given the forcing sequence $\{\tau_t\}_{t=0}^{\infty}$.

In general, heterogenous-agent models such as that described above generate an infinite-state distribution of agent types, as all individual characteristics (i.e., agents' wealth and implied optimal choices) depend on the personal history of each agent. Our approach here is to look for closed-form solutions characterised by finite numbers of households types and wealth states. The simplest class of equilibria that we consider has exactly two wealth states and four types of agents; we take it as a benchmark and analyse it extensively in the next two Sections. We subsequently extend our study to allow for more wealth states and agent types. In all of the cases, the derivation of the relevant class of equilibria involves three steps: first, we conjecture the general shape of the solution; second, we identify the conditions under which the hypothesised solution results; and third, we set the relevant parameters (here the productivity of home production) so that these conditions always hold along the equilibrium under consideration.

3 Equilibria with two-state wealth distributions

In this Section we construct the simplest class of equilibria with cross-sectional heterogeneity: that where the distribution of money holdings is two-state. In a nutshell, such equilibria have the following characteristics: all currently employed households hold the same amount of cash at the end of the period, which they hoard to self-insure against unemployment risk; all currently unemployed households, on the other hand, face a binding borrowing constraint and thus end the period with no cash. Since each household's type depends on both their beginning-of-period and end-of-period wealth, it follows that households can be of *four* different types only, depending on their employment status in the current period (on which end-of-period balances depend) and that in the period before (which determines their beginning-of-period balances). In short, while asymptotically no two agents have the same histories of idiosyncratic income shocks, the length of personal history which determines their type is of two periods only.

3.1 Conjectured equilibrium

We conjecture the existence of an equilibrium along which

$$\varphi_t = 0 \text{ if } \chi_t^i = 1 \text{ and } m_t^i = 0 \text{ if } \chi_t^i = 0, \tag{11}$$

that is, one where no employed households are borrowing-constrained (so that they all store cash to smooth consumption), while all unemployed households are borrowing-constrained (and thus hold no cash). Consider first the consumption level of an unemployed household under this conjecture. If this household was employed in the previous period then, from (4) and (11), their current consumption is:

$$c_t^i = m_{t-1}^i / (1 + \pi_t) + \delta + \gamma_t \ (> 0), \tag{12}$$

On the other hand, from (4) the consumption level of unemployed households who were already unemployed in the previous period is identical across such households and given by:

$$c_t^{uu} = \gamma_t + \delta \ (> 0). \tag{13}$$

We now turn to employed households. From (6) and (7), their consumption level is identical across employed households and independent of aggregate history, i.e.,

$$c^e = u'^{-1}(\phi) \ (> 0). \tag{14}$$

From equations (6)–(8) and (11), the intertemporal optimality condition for an employed household is $\phi = \beta E_t \left(\eta_{t+1}^i / (1 + \pi_{t+1}) \right)$. If this household is employed in the following period, which occurs with probability α, then $\eta_{t+1}^i = \phi$ (see (7)). If the household moves into unemployment in the next period, then from (6) $\eta_{t+1}^i = u'\left(c_{t+1}^i\right)$, where by construction c_{t+1}^i is given by (12). The Euler equation for employed households is thus:

$$\phi = \alpha\beta E_t \left(\frac{\phi}{1 + \pi_{t+1}} \right) + (1 - \alpha)\beta E_t \left(u'\left(\frac{m_t^i}{1 + \pi_{t+1}} + \delta + \gamma_{t+1} \right) \times \frac{1}{1 + \pi_{t+1}} \right). \tag{15}$$

Equation (15) summarises how persistent money shocks may affect real money demand by employed households. First, such shocks raise future inflation, π_{t+1}. On the one hand, this lowers the expected return on cash holdings, $1/(1+\pi_{t+1})$, and thus decreases real money demand, m_t^i; this is the usual intertemporal substitution effect of the real interest rate on savings. On the other hand, high future inflation lowers future consumption in case of a bad unemployment shock (see the $m_t^i/(1+\pi_{t+1})$ term in (15)), to which the agent may respond by *raising* real money demand; this is the income effect generated by changes in the real interest rate. Under our maintained assumption that consumption and leisure are not gross complements, the intertemporal substitution effect dominates for any level of unemployment risk and future inflation always lowers real money demand. Finally, persistent money shocks raise the expected transfer γ_{t+1}, which increases future consumption in case of bad idiosyncratic shock; this again goes towards lowering the need for self-insurance and thus the demand for real balances.

Since equation (15) relates individual money demands, m_t^i, to *aggregate* variables only, it implies that all employed households wish to hold the same quantity of real balances, which we now denote by m_t^e. We may thus rewrite equation (12), giving the consumption level of unemployed households which were previously employed, as follows:

$$c_t^{eu} = m_{t-1}^e/(1+\pi_t) + \delta + \gamma_t. \tag{16}$$

The labour supplies of employed households depend on whether they were employed or not in the previous period. Using Eqs. (4) and (14), these are respectively given by:

$$l_t^{ee} = c^e + m_t^e - m_{t-1}^e/(1+\pi_t) - \gamma_t, \tag{17}$$

$$l_t^{ue} = c^e + m_t^e - \gamma_t. \tag{18}$$

In other words, when all unemployed households are borrowing-constrained and no employed household is, households can be of four different types, depending only on their current and past employment statuses, with their personal history before $t-1$ being irrelevant. This distributional simplification results from the joint assumption that all unemployed households liquidate their asset holdings (i.e., $\forall i \in [0,1]$, $\chi_t^i = 0 \Rightarrow m_t^i = 0$), while all employed households choose the same levels of consumption and asset holdings due to linear disutility of labour (i.e., $\forall i \in [0,1]$, $\chi_t^i = 1 \Rightarrow m_t^i = m_t^e$).[4] We denote these four household

[4] Note that the property that working households all wish to hold the same quantity of end-of-period real balances, thanks to quasi-linear preferences and unbounded labour endowment, is also in Berentsen *et al.* (2005). In our model, however, only a subclass of households (the employed) are allowed to work and replete their money wealth at the end of each period, while *all* agents are in theirs.

types ee, eu, ue and uu, where the first and second letters refer to the date $t-1$ and date t employment statuses, respectively. Since our focus is on the way in which idiosyncratic unemployment risk affects self-insurance by the employed, we consider the effect of changes in α taking U in (1) as given (the implied probability of leaving unemployment is thus $\rho = 1-(1-\alpha)(1-U)/U$). We then write the asymptotic shares of household types as:

$$\omega^{ee} = \alpha(1-U), \; \omega^{eu} = \omega^{ue} = (1-\alpha)(1-U), \; \omega^{uu} = U-(1-\alpha)(1-U), \tag{19}$$

and we abstract from transitional issues regarding the distribution of household types by assuming that the economy starts at this invariant distribution.

Given the consumption and labour supply levels of each of household type, goods-market clearing now implies that:

$$\omega^{ee} l_t^{ee} + \omega^{ue} l_t^{ue} + U\delta = (1-U)c^e + \omega^{eu} c_t^{eu} + \omega^{uu} c_t^{uu}. \tag{20}$$

In the equilibrium under consideration, which we assume to prevail from date 0 onwards, unemployed households hold no money while all employed households hold the real quantity m_t^e. Money-market clearing thus requires that

$$(1-U)\, m_t^e = m_t. \tag{21}$$

3.2 Existence conditions

The condition for the distribution above to be an equilibrium is that the borrowing constraint does not bind for ee and ue households but always binds for both uu and eu households. The constraint is not binding for employed households if the latter never wish to borrow. Thus, interior solutions to (15) must always be such that:

$$m_t^e \geq 0. \tag{22}$$

On the other hand, the Lagrange multiplier φ_t^i has to be positive when households are unemployed, so that from (6)-(8) we must have $\eta_t^i > \beta E_t \eta_{t+1}^i / (1+\pi_{t+1})$. First consider uu households, whose current consumption is just $\delta + \gamma_t$ (see (13)), and thus for whom $\eta_t^i = u'(\delta + \gamma_t)$. These households remain unemployed with probability ρ, in which case they will also consume $\delta + \gamma_t$ in the following period and thus $\eta_{t+1}^i = u'\left(\delta + \gamma_{t+1}\right)$. They leave unemployment with probability $1-\rho$ and will then consume $u'^{-1}(\phi)$ in the following period, so that $\eta_{t+1}^i = u'(u'^{-1}(\phi)) = \phi$. Thus, uu households are borrowing-constrained whenever:

$$u'\left(\delta+\gamma_t\right) > \rho\beta E_t\left(\frac{u'\left(\delta+\gamma_{t+1}\right)}{1+\pi_{t+1}}\right)+\left(1-\rho\right)\beta E_t\left(\frac{\phi}{1+\pi_{t+1}}\right). \tag{23}$$

We now turn to eu households. Their current consumption is given by Eq. (16), so that $\eta_t^i = u'\left(m_{t-1}^e/\left(1+\pi_t\right)+\delta+\gamma_t\right)$, while, just like uu households, they will be either uu or ue households in the following period. Thus, eu households are borrowing-constrained whenever:

$$u'\left(\frac{m_{t-1}^e}{1+\pi_t}+\delta+\gamma_t\right) > \rho\beta E_t\left(\frac{u'\left(\delta+\gamma_{t+1}\right)}{1+\pi_{t+1}}\right)+\left(1-\rho\right)\beta E_t\left(\frac{\phi}{1+\pi_{t+1}}\right). \tag{24}$$

If (22) holds then (24) is more stringent than (23), so (24) is a sufficient condition for both uu and eu households to be borrowing-constrained. We show in Appendix A that when δ lies in a range (δ_-, δ_+), where $0 \leq \delta_- < \delta_+$, then both (22) and (24) hold for all $t \geq 0$, provided that aggregate shocks have sufficiently small support. Intuitively, for our equilibrium to exist home production must be sufficiently productive to deter unemployed households from saving, whilst at the same time being sufficiently unproductive to induce positive precautionary savings by employed households.

3.3 Equilibrium dynamics

We are now in a position to derive the solution dynamics of our equilibrium with limited wealth heterogeneity. Using (2), (3), (15) and (21), we can summarise the dynamic behaviour of the economy by a single forward-looking equation, namely,

$$m_t^e = \alpha\beta E_t\left(\frac{m_{t+1}^e}{\tau_{t+1}}\right)+\frac{\left(1-\alpha\right)\beta}{\phi}E_t\left(\frac{m_{t+1}^e}{\tau_{t+1}}u'\left(\delta+\frac{m_{t+1}^e}{\tau_{t+1}}\left(U+\left(1-U\right)\tau_{t+1}\right)\right)\right). \tag{25}$$

Equation (25) determines the equilibrium dynamics of the real money balances held by employed households, $\{m_t^e\}_{t=0}^{\infty}$, as a function of the money growth sequence $\{\tau_t\}_{t=0}^{\infty}$, the exogenous state variable here. Finally, we assume that τ_t has small bounded support $[\tau_-, \tau_+]$, with $\tau_- > \beta$.

All equilibrium variables at date t can be expressed as functions of m_t^e and τ_t only. For example, substituting (2)–(3) and (21) into (17)–(18), we derive the following expressions for the labour supplies of employed households, depending on their specific type:

$$l_t^{ee} = c^e + Um_t^e\left(\frac{\tau_t-1}{\tau_t}\right),\ l_t^{ue} = c^e + m_t^e\left(\frac{1+U\left(\tau_t-1\right)}{\tau_t}\right), \tag{26}$$

where $l_t^{ee}, l_t^{ue} > 0$. Similarly, substituting (2)–(3) and (21) into (13) and (16), the consumption levels of unemployed households can be expressed as:

$$c_t^{eu} = \delta + m_t^e \left(\frac{U + (1-U)\tau_t}{\tau_t} \right), \ c_t^{uu} = \delta + (1-U)\, m_t^e \left(\frac{\tau_t - 1}{\tau_t} \right). \tag{27}$$

It is apparent from equations (26)–(27) that money growth shocks can affect real variables either directly through changes in τ_t (inside each bracketed term), or indirectly through their potential effect on current real money demand, m_t^e (as given by equation (25)). For reasons that will become clear shortly, we refer to the former and the latter channels as the 'intratemporal' and 'intertemporal' inflation taxes, respectively.

4 Intratemporal vs. intertemporal effects of monetary shocks and inflation taxes

4.1 The intratemporal inflation tax

The intratemporal effects of monetary shocks can be pinpointed in the case where money growth is i.i.d., so that the current shock does not alter households' expectations about future inflation. With i.i.d. aggregate shocks, the first-order approximation to equation (25) yields the following constant path for m_t^e:[5]

$$m_t^e = m^e = \frac{1+\pi}{1+(1-U)\pi} \left(u'^{-1} \left(\frac{(1+\pi-\alpha\beta)\,\phi}{(1-\alpha)\,\beta} \right) - \delta \right), \tag{28}$$

where unindexed variables denote steady-state values (all of which are summarised in Appendix A). Two properties of m^e are worth mentioning at this point. First, m^e falls with α, as less idiosyncratic unemployment risk reduces employed households' incentives to self-insure against this risk. Second, under our maintained assumption that $\sigma(c) \leq 1$, m^e falls with π (this is proved in Appendix A): as inflation increases, the return to holding real balances falls and money becomes less valuable as a self-insurance device against idiosyncratic unemployment shocks.

We can now turn to the effects of i.i.d. aggregate shocks on labour supply and output. Equations (3) and (28) imply that $\pi_t = \tau_t - 1$. The individual labour supplies are thus:

$$l_t^{ee} = c^e + m^e \left(\frac{U\pi_t}{1+\pi_t} \right), \ l_t^{ue} = c^e + m^e \left(\frac{1+U\pi_t}{1+\pi_t} \right). \tag{29}$$

[5] This can be checked by linearising (25) around steady-state real balances and money growth, (m^e, τ), and solving the resulting equation forwards. That $\sigma(c) \leq 1$ implies that the equilibrium is unique and non-cyclical, while i.i.d shocks preclude time-variations in real balances.

The latter equations summarise the redistributive effects of the current inflation tax on labour supplies. Note that l_t^{ee} rises but l_t^{ue} falls with π_t, following households' adjustments to the intratemporal wealth redistribution generated by inflation. After an inflation shock, the households who pay the inflation tax in period t are those who hold money at the beginning of period t (ee and eu households), while the households who benefit from the corresponding inflation subsidy are those who do not hold money at the beginning of period t (ue and uu households). Consequently, ee households are hurt by the shock and increase their labour supply to maintain their desired levels of consumption and money wealth, while ue households can afford to work less than they would have done had the shock not occurred. From equations (19) and (29), market output, $\omega^{ee} l_t^{ee} + \omega^{ue} l_t^{ue}$, is given by:

$$y_t = (1 - U)\, c^e + (1 - U)\, m^e \left(\frac{U\pi_t + 1 - \alpha}{1 + \pi_t} \right). \tag{30}$$

Market output increases with current inflation (i.e. greater labour supply by ee households dominates the reduced labour supply of ue households) provided that $U + \alpha > 1$, or equivalently from (1), that $\alpha + \rho > 1$ (that is, the average persistence in employment status must be sufficiently high). For small shocks, equation (30) can be approximated by the following linear 'output-inflation tradeoff' relation:

$$y_t = y + \mu\, (\pi_t - \pi)\,, \tag{31}$$

where

$$\mu = \frac{U + \alpha - 1}{(1 + \pi)\,(1 + (1 - U)\,\pi)} \left(u'^{-1} \left(\frac{(1 + \pi - \alpha\beta)\,\phi}{(1 - \alpha)\,\beta} \right) - \delta \right). \tag{32}$$

This tradeoff equation is reminiscent of those derived by Lucas (1973) or Ball *et al.* (1988); however, the underlying mechanism that we emphasise here works very differently. In Lucas, agents raise production after an inflationary money shock because they cannot fully disentangle changes in relative prices from variations in the general price level; in Ball *et al.*, the output-inflation tradeoff arises naturally from nominal rigidities. In contrast, our model here features perfect information and fully-flexible prices, but heterogenous cash balances. Consequently, lump-sum monetary injections redistribute wealth from cash-rich to cash-poor households, thereby inducing employed households to alter their labour supplies in order to offset the implied wealth effects. Interestingly, the model predicts that higher trend inflation lowers the impact of inflation shocks on output (i.e., $\partial\mu/\partial\pi < 0$), because it lowers money holdings by employed households and thus mitigates the redistributive effects of these shocks. We may thus conclude that this negative relation is perfectly consistent with price flexibility, contrary to the claim in Ball *et al.* (1988) that it supports the hypothesis of nominal rigidities. We summarise the results obtained so far in the following proposition:

Proposition 1. Steady-state real money holdings by employed households, m^e, increase with idiosyncratic unemployment risk, $1-\alpha$, and decrease with mean inflation, π. With i.i.d. money-growth shocks, a necessary and sufficient condition for shocks to raise current output is $U+\alpha>1$ (or, equivalently, $\alpha+\rho>1$), while the effect of the shock on output is stronger the lower is mean inflation.

4.2 The intertemporal inflation tax

Central to the transmission of monetary shocks here is the rôle, and determinants, of the real money holdings held by the employed as a buffer against idiosyncratic unemployment risk. Under i.i.d. money growth shocks, these holdings are constant over time as they are immediately and entirely repleted by employed households (through variations in labour supply) following a shock that redistributes current wealth. Obviously, this simple adjustment to exogenous disturbances is more complicated if current real money demand is itself affected by changes in expected inflation, which is precisely what occurs when money growth shocks display persistence. In order to examine the effect of auto-correlated aggregate shocks, assume that money growth obeys the following AR(1) process:

$$\tau_t = (1-\chi)\,\tau + \chi\tau_{t-1} + \epsilon_t, \tag{33}$$

where $\chi \in (0,1)$ and $\{\epsilon_t\}_{t=0}^{\infty}$ is a white-noise process with zero mean and small bounded support. Linearising (25) around the steady state, we obtain:

$$\hat{m}_t^e = AE_t\left(\hat{m}_{t+1}^e\right) - BE_t\left(\hat{\tau}_{t+1}\right), \tag{34}$$

where the hatted values denote proportional deviations from steady state (e.g., $\hat{m}_t^e = \left(m_t^e - m^e\right)/m^e$) and A and B are the following constants:

$$A \equiv 1 - \frac{(1+\pi-\alpha\beta)\left(1-\delta/c^{eu}\right)\sigma\left(c^{eu}\right)}{1+\pi} \in (0,1)\,,$$
$$B \equiv 1 - \frac{(1+\pi-\alpha\beta)\left(1-\delta/c^{eu}\right)\sigma\left(c^{eu}\right)}{1+\pi}\left(\frac{U}{1+(1-U)\,\pi}\right) \in (A,1)\,.$$

That $A \in (0,1)$ implies that the equilibrium path generated by (33)–(34) is unique (i.e., determinate); then, iterating (34) forwards under the transversality condition (10) yields:

$$\hat{m}_t^e = -\left(\frac{B\chi}{1-A\chi}\right)\hat{\tau}_t, \tag{35}$$

where $B\chi/(1-A\chi) > 0$. Equation (35) summarises the effect of current money growth on current real balances working through changes in future money growth (both relative to the steady state). With autocorrelated shocks, a relatively high current inflation tax on employed households signals high future inflation, thereby reducing the desirability of money as a means of self-insurance and households' incentives to supply labour to acquire it. To see how such money demand adjustments alter the response of market output to monetary shocks, use equations (2), (21) and (17)–(18) again to write (30) in the following slightly more general form:

$$y_t = (1-U)\, c^e + (1-U)\, m_t^e \left(\frac{U(\tau_t - 1) + 1 - \alpha}{\tau_t} \right). \tag{36}$$

Persistent money growth shocks lower m_t^e (because of the future inflation taxes on real money), but raise $(U(\tau_t - 1) + 1 - \alpha)/\tau_t$ provided that $U + \alpha > 1$ (through contemporaneous wealth redistribution). In other words, the effect of future expected inflation on real money demand runs counter the effect induced by intratemporal wealth redistribution. Linearising equation (36) and using (35), we obtain the following results.

Proposition 2. Assume that money growth, τ_t, follows an AR(1) process with autocorrelation parameter $\chi \in (0,1)$. Then, the higher is χ, the lower is the impact of monetary shocks on output, while a necessary and sufficient condition for these shocks to raise output is:

$$\frac{U + \alpha - 1}{U\pi + 1 - \alpha} > \frac{B\chi}{1 - A\chi}$$

Whether the latter condition holds or not ultimately depends on the deep parameters that enter both sides of the inequality. When $\chi \to 0$, the analysis of the previous Section applies and monetary shocks raise current output if and only if $U + \alpha > 1$. However, as χ rises the intertemporal effect becomes larger and reduces the impact of shocks, possibly (but not necessarily) leading to a reversal in the sign of the tradeoff for large values of χ. Finally, it is straightforward to show that a sufficiently high value of mean inflation always lead to the violation of this condition (and thus to a reversal in the tradeoff slope), as it tends to mitigate the current inflation tax (relative to expected inflation taxes) following a persistent monetary shock.

Panel A of Figure 1 illustrates the dynamic effects of a money growth shock on monetary and aggregate supply variables. We set $\alpha = 0.95$, $\beta = 0.98$, $\chi = 0.6$ and $\tau = 1$. We interpret U as the share of borrowing-constrained households in the economy and set it to 0.2 (e.g., Jappelli, 1990). The instantaneous utility function is $\ln c - l$ and the home production parameter $\delta = 0.9\delta_+$ (these parameters ensure that the existence conditions stated in Section 3.2 are satisfied.)

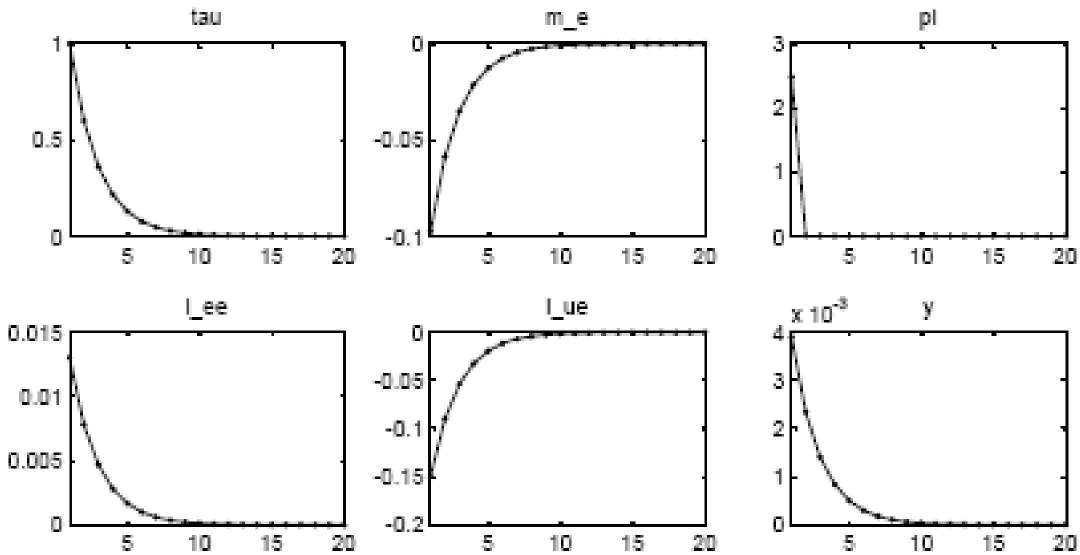
Following a persistent shock, real money demand falls (due to the future inflation taxes expected by employed households), and inflation jumps up. *ee* households persistently raise their labour supply to offset the inflation tax, while *ue* households reduce their labour supply in response to the inflation subsidy. Overall, the responses of labour supplies to the shock imply that the latter persistently raises output; this is because the labour supply response of *ue* households, although relatively large at the individual level, is actually small in the aggregate (with $\alpha = 0.95$, equation (19) implies that there are about 20 times as many *ee* households as *ue* households.)

4.3 Asymmetric money injections

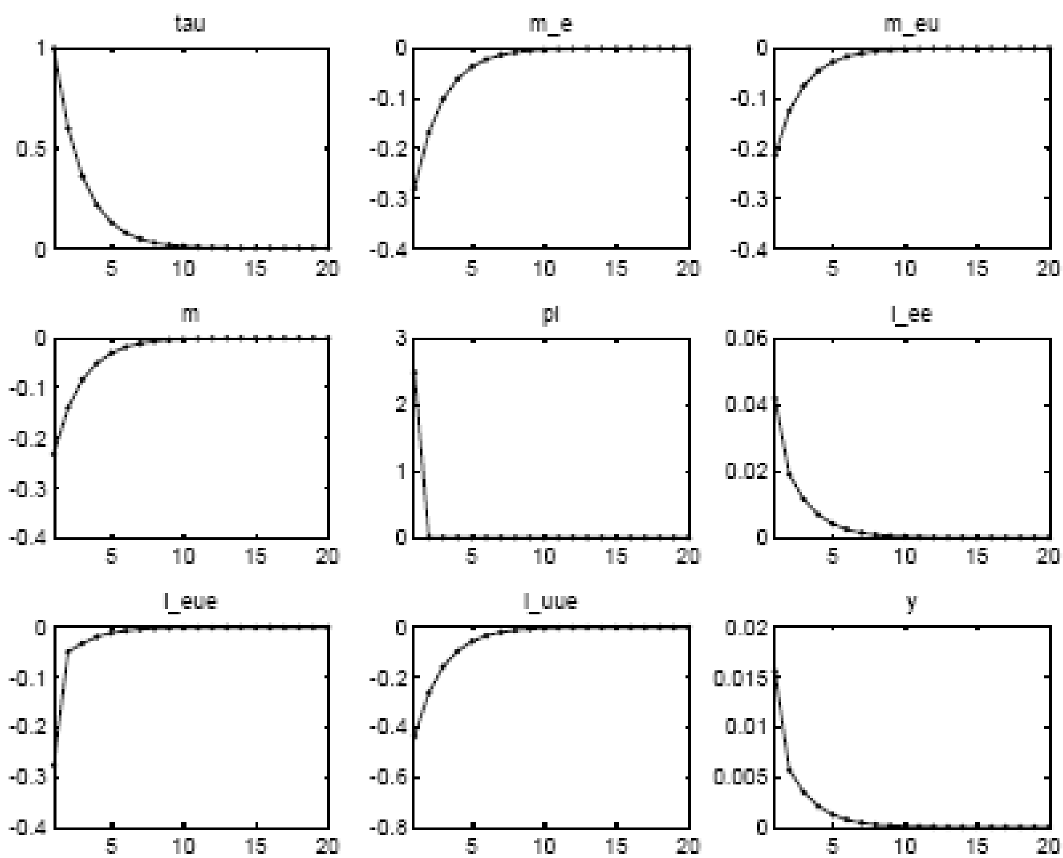
In order to analyse how expected inflation taxes affect households' optimal responses to the current shock, we now study the distributional and aggregate implications of *asymmetric* money injections. We can think of a number of reasons why money injections would be biased towards some particular agents, rather than being symmetric. If the government uses fiat money to finance a public unemployment insurance scheme, then unemployed households as a group will receive more newly-issued money than what their share in the economy would imply. Conversely, if money injections include a proportional component, then initially cash-rich (i.e., previously employed) agents will receive more newly-printed money than under the symmetric case.

In the context of our four-agent model, the simplest kind of asymmetric injection is that where households are distinguished on the basis of their current employment status.[6] We thus assume that unemployed households receive a real injection $\gamma_t^u = a\gamma_t$, where $a \in [0, 1/U]$ and γ_t is the aggregate injection defined by (2); by implication, employed households receive $\gamma_t^e = b\gamma_t$, where $b \equiv (1 - aU) / (1 - U)$ (so that $U\gamma_t^u + (1 - U)\gamma_t^e = \gamma_t$). Injections are biased towards the unemployed (employed) when $a > 1$ (< 1). When $a = 0$, all newly-issued money is handed out to the employed, while when $a = 1/U$ it entirely benefits the unemployed.

[6] We obtain close results when discrimination takes place over beginning-of-period wealth, rather than employment status. We focus on the latter here because it is arguably more easily observable by the government than the former.



A. Equilibrium with two wealth states



B. Equilibrium with three wealth states

Figure 1. Level-deviations from the steady state of money growth, real money holdings, equilibrium real balances (in the three-wealth state case), inflation, labour supplies and output, following a normalised, unexpected money growth innovation.

We expect such asymmetries to alter both the intratemporal and the intertemporal non-neutralities at work in the symmetric case. Consider the case where $a > 1$ (the reverse reasoning applies when $a < 1$). On the one hand, this should generate a greater output response to a one-off money shock, as the intratemporal inflation tax paid by the employed is magnified, relative to the case where $a = 1$. On the other hand, such biased injections, inasmuch that they are *expected* to occur in the future when money shocks are persistent, should *lower* the need for self-insurance by the employed and thus lower money wealth inequalities and the implied impact of redistributive shocks. Overall, the way in which asymmetric injections modify the output response to money shocks relative to the symmetric case will depend on the relative strength of these two effects.[7]

Let us start with the intratemporal effect. First, we can compute output again, $y_t = (1-U)\,c^e + \omega^{ee} l_t^{ee} + \omega^{ue} l_t^{ue}$, taking account of the fact that in (17)–(18) the real injection is $b\gamma_t$ rather than γ_t. Then, using (2) and (21) and rearranging, we have:

$$y_t = (1-U)\,c^e + (1-U)\,m_t^e\left(\frac{aU(\tau_t - 1) + 1 - \alpha}{\tau_t}\right), \tag{37}$$

which is a generalisation of Eq. (36) allowing for biased money injections. Holding m_t^e constant (i.e., assuming i.i.d. money growth shocks), a value of a that is greater (less) than 1 *raises* (lowers) the impact response of output to the shock. This is because this value of a is associated with a higher (lower) inflation tax paid by the employed, to which they respond by increasing their labour supply more (less) than in the symmetric case. The effect on output is maximal at $a = 1/U$, while output is unaffected by money injections when $a = 0$ (i.e., there is *no* inflation tax paid by the rich when they receive all the newly-issued money.)

Let us know consider the intertemporal effect, working through the dynamic adjustments in m_t^e that are induced by persistent money growth shocks. First, take (15) with $a\gamma_{t+1}$ instead of γ_{t+1}, use (2), (3) and (21) and rearrange to obtain the following dynamics for real money

[7] It is easy to show that the conditions for the existence of a four-agent equilibrium in the asymmetric-injections case are similar to those in the symmetric case; namely, there always exists a range (δ_-, δ_+), with $\delta_- < \delta_+$, such that if $\delta \in (\delta_-, \delta_+)$ then employed households are never constrained while unemployed households always are. More specifically, from Eq. (38), steady state real money holdings by the employed for any feasible value of a are:

$$m^e = \frac{(1+\pi)}{1 + a(1-U)\pi}\left(u'^{-1}\left(\frac{\phi(1+\pi-\alpha\beta)}{(1-\alpha)\beta}\right) - \delta\right),$$

and thus $m^e > 0$ under condition (A2) in Appendix A. Combining (A2) with the requirement that *eu* households be borrowing constrained gives condition (A5) which, as in the symmetric-injections case, always holds for $\pi > \beta - 1$.

demand :

$$m_t^e = \alpha\beta E_t\left(\frac{m_{t+1}^e}{\tau_{t+1}}\right) + \frac{(1-\alpha)\beta}{\phi}E_t\left(\frac{m_{t+1}^e}{\tau_{t+1}}u'\left(\frac{m_{t+1}^e}{\tau_{t+1}}\left(1+a\left(1-U\right)\left(\tau_{t+1}-1\right)\right)+\delta\right)\right), \quad (38)$$

which generalises Eq. (25). Then, linearising (38) and iterating the resulting expression forwards as before, we find that the impact response of current real money demand by the employed, as a proportional deviation from the steady state, to a persistent real money growth shock is given by:

$$\hat{m}_t^e = -\left(\frac{B(a)\chi}{1-A\chi}\right)\hat{\tau}_t, \quad (39)$$

where A is as in Section 4.2 and B now depends on a as follows:

$$B(a) \equiv 1 - \frac{\sigma\left(1+\pi-\alpha\beta\right)\left(1-a\left(1-U\right)\right)\left(1-\delta/c^{eu}\right)}{\left(1+\pi\right)\left(1+a\left(1-U\right)\pi\right)}.$$

As in the symmetric-injections case, the fall in the demand for real money balances by employed households is all the more pronounced that money shocks are persistent (i.e., χ is large). The fact that $\partial B(a)/\partial a > 0$ means that large values of a magnify this effect even more: as the bias in favour of the unemployed increases, the demand for self-insurance by employed households falls for any given value of χ, thereby leading to a larger drop in their real money demand. This effect will work to reduce the response of y_t to a τ_t-shock (see (37)) as less inequalities in money holdings mitigate the impact of money shocks. Linearising (37) and using (39), we find that under asymmetric injections the necessary and sufficient condition for money shocks to raise output is:

$$\frac{aU+\alpha-1}{aU\pi+1-\alpha} > \frac{B(a)\chi}{1-A\chi},$$

which may be compared to the inequality stated in proposition 2 above. Here the two effects just discussed are reflected in the fact an increase in a raises both sides of the inequality, so it is in general ambiguous whether it is higher or lower values of a, holding other parameters constant, that are more likely to lead to a reversal in the slope of the tradeoff.

Importantly, the way money injections are transmitted to nominal prices and inflation over time depends on the primary beneficiaries of such injections. Substituting (39) into the linearised versions of (3) and using (21), one finds the following approximation to current inflation:

$$\pi_t \simeq \pi + \hat{\tau}_t - \hat{m}_t^e + \hat{m}_{t-1}^e = \pi + C\left(a\right)\hat{\tau}_t + D\left(a\right)\hat{\tau}_{t-1},$$

where

$$C\left(a\right) \equiv \frac{1+\left(B\left(a\right)-A\right)\chi}{1-A\chi} \quad \text{and} \quad D\left(a\right) \equiv -\frac{B\left(a\right)\chi}{1-A\chi}.$$

The coefficient $C(a)$ (≥ 0) measures the impact effect of a money shock on inflation; after one period, we have that $\hat{\tau}_t = \chi \hat{\tau}_{t-1}$ and thus $\pi_t \simeq \pi + (C(a)\chi + D(a))\hat{\tau}_{t-1}$. $C(a)$ (≥ 0) is increasing in a whereas $C(a)\chi + D(a)$ (≥ 0) is decreasing in a; in other words, the larger is the bias towards the unemployed, the larger is the *immediate* impact of the shock on inflation and the smaller its *delayed* effect on inflation (for any given positive value of χ). This is the case because unemployed households are borrowing-constrained and spend all their additional nominal income in the current period; the new money they are given is thus immediately reflected into changes in the nominal price of goods. On the contrary, employed households tend to *save* some of their additional nominal income for self-insurance purposes, rather than spend it all on goods, leading to a delayed effect on nominal prices. Overall, money shocks are transmitted into nominal prices and inflation all the more quickly that the bias in favour of the unemployed rises.

4.4 Welfare considerations

Since the non-neutrality mechanism described in this paper relies on wealth redistribution (both at the time of the shock and in the future), it directly affects the welfare of every single agent. There obviously are potential losers and winners from this wealth redistribution, meaning that we should not in general expect monetary shocks to unambiguously produce better or worse dynamic equilibria in the Pareto sense. For the sake of simplicity, we here explore such welfare effects in the context of symmetric money injections.

To analyse these welfare effects further, recall from (26)–(27) and (34) that all time-t variables can be expressed as functions of the only state variable of the model, current money growth τ_t, and that τ_{t+1} only depends on τ_t. We focus on the dynamic and welfare effects of an once-off, unexpected aggregate shock, and call $W^i(\tau_t)$ the maximum value function of agent i when current money growth is τ_t:

$$\begin{aligned} W^i(\tau_t) &= E\left(\sum\nolimits_{k=0}^{\infty} \beta^k \left(u(c^i(\tau_{t+k})) - \phi l^i(\tau_{t+k})\right) \Big| \chi_t^i, \tau_t\right) \\ &= u(c^i(\tau_t)) - \phi l^i(\tau_t) + \beta E\left(W^i(\tau_{t+1}) \left| \chi_t^i, \tau_t\right.\right) \end{aligned}$$

Let $W_\tau^i = \partial W^i(\tau_t)/\partial \tau_t|_{\tau_t=\tau}$ be the first derivatives of this value function, evaluated at the steady state, and recall from (33) that $\partial \tau_{t+1}/\partial \tau_t = \chi$. Then, in the vicinity of the steady state (i.e., where $\partial W^i(\tau_{t+1})/\partial \tau_{t+1} \simeq \partial W^i(\tau_t)/\partial \tau_t = W_\tau^i$), and given the transition probabilities across employment statuses, the first derivatives of the Bellman equations for each agent type are related as follows:

$$\begin{aligned}
W_{\tau}^{ee} &= -\phi \left.\frac{\partial l^{ee}(\tau_t)}{\partial \tau_t}\right|_{\tau_t=\tau} + \alpha\beta\chi W_{\tau}^{ee} + (1-\alpha)\beta\chi W_{\tau}^{eu}, \\
W_{\tau}^{ue} &= -\phi \left.\frac{\partial l^{ue}(\tau_t)}{\partial \tau_t}\right|_{\tau_t=\tau} + \alpha\beta\chi W_{\tau}^{ee} + (1-\alpha)\beta\chi W_{\tau}^{eu}, \\
W_{\tau}^{eu} &= \left.\frac{\partial u(c^{eu}(\tau_t))}{\partial \tau_t}\right|_{\tau_t=\tau} + \rho\beta\chi W_{\tau}^{uu} + (1-\rho)\beta\chi W_{\tau}^{ue}, \\
W_{\tau}^{uu} &= \left.\frac{\partial u(c^{uu}(\tau_t))}{\partial \tau_t}\right|_{\tau_t=\tau} + \rho\beta\chi W_{\tau}^{uu} + (1-\rho)\beta\chi W_{\tau}^{ue}.
\end{aligned}$$

The solution to this system expresses the first derivatives of the four value functions as (cumbersome) functions of the deep parameters of the model.[8] The effects of money growth shocks on the intertemporal utility of individual households can easily be computed when χ is small. From the above system and equations (26)–(28), when $\chi \to 0$ (the i.i.d. case in the limit) we have:

$$W_{\tau}^{ee} \to -m^e \phi U/\tau^2 < 0,\ W_{\tau}^{ue} \to m^e \phi (1-U)/\tau^2 > 0,$$
$$W_{\tau}^{eu} \to -m^e U u'(c^{eu})/\tau^2 < 0,\ W_{\tau}^{uu} \to m^e (1-U) u'(c^{uu})/\tau^2 > 0.$$

These latter equations state that utility always rises for households who benefit from the inflation subsidy at the time of the shock (uu and ue households, i.e., those who hold no cash at the beginning of the period), while those who pay for the inflation tax (ee and eu households, who are cash-rich at the beginning of the period) necessarily experience a welfare loss. We cannot derive such clear-cut results on individuals' welfare, however, outside of this limiting case, due to the combined effect of future expected redistribution and the transition of households between employment states. For example, a households which currently suffers from the inflation tax (say, a ee household) may expect to benefit from it in the future (if, for example, α is low relative to ρ), making the overall welfare of this household a priori ambiguous. (Conversely, a household which currently benefits from the redistributive effect of inflation may suffer from it in the future if the probability of being cash-rich for sufficiently many periods in the future is high).

[8] More specifically, the solution to this system expresses the W_{τ}^{i} as functions of the $\partial(.)/\partial\tau_t|_{\tau_t=\tau}$ terms, which latter can in turn be computed from equations (26)–(27) and (34)

5 Equilibria with n-state wealth distributions

Sections 3 and 4 have focused on the simplest possible cross-sectional wealth distribution, with immediate asset liquidation by households who became unemployed. In this Section we introduce richer distributions by letting households only gradually deplete their money holding when unemployed. We start by explicitly constructing the class of three-wealth state equilibria, and then describe more informally how the model can be generalised to handle n-state cross-sectional distributions.

5.1 3 wealth states

Our conjectured equilibrium here is one in which households falling into unemployment take two periods to entirely liquidate their real balances (i.e., partial liquidation occurs in the first period of unemployment). Just as in the two-wealth state case, all employed households hold the same quantity of real money, denoted m_t^e, at the end of the current period, regardless of their employment history. Unemployed households hold a quantity of money m_t^{eu} at the end of the first unemployment period and 0 at the end of the second unemployment period. This equilibrium thus adds an additional layer of personal history which determines the household types, relative to the two-state case: unemployed households' money holdings now depend on both the current and the two previous employment states, and hence so will the labour supplies of employed households.

Formally, letting φ_t^i be the Lagrange multiplier on the borrowing constraint faced by household i, our conjectured equilibrium is such that:

$$\varphi_t^i = 0 \text{ if } \chi_t^i = 1, \;\; \varphi_t^i = 0 \text{ if } \left(\chi_t^i, \chi_{t-1}^i\right) = (0,1)\,, \;\text{ and } m_t^i = 0 \text{ if } \left(\chi_t^i, \chi_{t-1}^i\right) = (0,0)$$

Budget constraints. We can easily recover the budget constraints faced by each household type in the economy under this conjecture, by proceeding in the same way as in Section 3.1 above for the two wealth states case. Provided that this equilibrium exists, it has exactly *six* types of agents, with (real) budget constraints:

$$ee \quad : \quad c^e + m_t^e = \frac{m_{t-1}^e}{1+\pi_t} + l_t^{ee} + \gamma_t \tag{40}$$

$$eue \quad : \quad c^e + m_t^e = l_t^{eue} + \frac{m_{t-1}^{eu}}{1+\pi_t} + \gamma_t \tag{41}$$

$$uue \quad : \quad c^e + m_t^e = l_t^{uue} + \gamma_t \tag{42}$$

$$eu \quad : \quad c_t^{eu} + m_t^{eu} = \frac{m_{t-1}^e}{1+\pi_t} + \gamma_t + \delta \tag{43}$$

$$euu \quad : \quad c_t^{euu} = \frac{m_{t-1}^{eu}}{1+\pi_t} + \gamma_t + \delta \tag{44}$$

$$uuu \quad : \quad c_t^{uuu} = \gamma_t + \delta \tag{45}$$

In short, there are now three (rather two) types of unemployed households: *eu* households, who were employed in the previous period, with beginning-of-period nominal balances of M_{t-1}^e (i.e., $m_t^e/(1+\pi_t)$ in real terms) and end-of-period nominal balances M_t^{eu} (m_t^{eu} in real terms); *euu* households, who became unemployed in the previous period, with beginning-of-period nominal balances of M_{t-1}^{eu} ($m_{t-1}^{eu}/(1+\pi_t)$ in real terms) and no end-of-period balances (asset liquidation is total for them after two periods); and *uuu* households, who enter and leave the period with no money wealth. By implication, there are also three types of employed households, since the latter adjust their labour supply according to the wealth that they inherited from their previous asset accumulation: *ee* households, who behave identically to the two-state case; *eue* households, who leave unemployment with some real balances (i.e., $m_{t-1}^{eu}/(1+\pi_t)$) left over from their previous employement, two periods beforehand; and *uue* households, who enter employment with zero beginning-of-period wealth. These households will supply l_t^{ee}, l_t^{eue} and l_t^{uue} units of labour at date t, respectively.

Asymptotic shares of households. Let ω be the column vector made of the asymptotic shares of household types, in the same order as that for the budget constraints above, and Ω the 6×6 transition matrix across *household types* generated by the transition probabilities across employment statuses. The invariant distribution of types satisfies $\omega'\Omega = \omega'$ which, after some algebra, yields :

$$\omega = \begin{bmatrix} \omega^{ee} \\ \omega^{eue} \\ \omega^{uue} \\ \omega^{eu} \\ \omega^{euu} \\ \omega^{uuu} \end{bmatrix} = \frac{1}{2-\alpha-\rho} \begin{bmatrix} \alpha(1-\rho) \\ (1-\alpha)(1-\rho)^2 \\ (1-\alpha)(1-\rho)\rho \\ (1-\alpha)(1-\rho) \\ (1-\alpha)(1-\rho)\rho \\ (1-\alpha)\rho^2 \end{bmatrix}.$$

Money demands. The two equations driving the dynamics of the model are: i) the Euler equation characterising the real money demand of employed households (i.e., ee, eue and uue households), who stay employed in the next period with probability α and become unemployed with complementary probability; and ii) the Euler equation characterising the real money demand of agents who have just become unemployed (i.e., eu households), who stay unemployed with probability ρ and move back into employment with complementary probability. From equations (43)–(44) and the fact that $u'(c^e) = \phi$ for employed households, the real money demands satisfy the following relationships:

$$\begin{aligned} \text{employed} \quad &: \quad \phi = \alpha\beta E_t\left(\frac{\phi}{1+\pi_{t+1}}\right) \\ &\quad + (1-\alpha)\beta E_t\left(u'\left(\frac{m_t^e}{1+\pi_{t+1}} - m_{t+1}^{eu} + \delta + \gamma_{t+1}\right)\frac{1}{1+\pi_{t+1}}\right); \qquad (46) \\ eu \quad &: \quad u'\left(\frac{m_{t-1}^e}{1+\pi_t} - m_t^{eu} + \delta + \gamma_t\right) = (1-\rho)\beta E_t\left(\frac{\phi}{1+\pi_{t+1}}\right) \\ &\quad + \rho\beta E_t\left(u'\left(\frac{m_t^{eu}}{1+\pi_{t+1}} + \delta + \gamma_{t+1}\right)\frac{1}{1+\pi_{t+1}}\right). \qquad (47) \end{aligned}$$

These two Euler equations will, together with the market clearing conditions that follow, drive the dynamics of the three wealth state equilbrium.

Market clearing. The only difference with the four-agent model here is that the market-clearing conditions now include a larger set of household types. More specifically, the clearing of the goods and money markets now require, respectively,

$$\omega^{ee} l_t^{ee} + \omega^{uue} l_t^{uue} + \omega^{eue} l_t^{eue} + U\delta = (1-U)\,c^e + \omega^{eu} c_t^{eu} + \omega^{euu} c_t^{eu} + \omega^{uuu} c_t^{uuu}, \qquad (48)$$

$$\omega^e m_t^e + \omega^{eu} m_t^{eu} = m_t. \qquad (49)$$

For future reference, we now substitute (49) into (2) and use (3) to rewrite real money injections and inflation as functions of the two endogenous state variables, m_t^e and m_t^{eu}, and the exogenous state variables, τ_t:

$$\gamma_t = (\omega^e m_t^e + \omega^{eu} m_t^{eu})\left(1 - \frac{1}{\tau_t}\right),\ 1+\pi_t = \left(\frac{\omega^e m_{t-1}^e + \omega^{eu} m_{t-1}^{eu}}{\omega^e m_t^e + \omega^{eu} m_t^{eu}}\right)\tau_t. \qquad (50)$$

Existence conditions. The existence conditions for this equilibrium are as follows. First, neither employed households nor eu households must be borrowing constrained, i.e. they must all be willing to save rather than borrow. It must thus be the case that, for all t,

$$m_t^e,\, m_t^{eu} \geq 0.$$

Second, both euu and uuu households must be borrowing-constrained in equilibrium, i.e., their marginal utility of current consumption must be higher than their expected, discounted marginal utility. We must thus have:

$$euu \quad : \quad u'\left(\frac{m_{t-1}^{eu}}{1+\pi_t}+\gamma_t+\delta\right) > (1-\rho)\,\beta E_t\left(\frac{\phi}{1+\pi_{t+1}}\right)+\rho\beta E_t\left(\frac{u'\left(\gamma_{t+1}+\delta\right)}{1+\pi_{t+1}}\right), \tag{51}$$

$$uuu \quad : \quad u'\left(\frac{u'\left(\gamma_t+\delta\right)}{1+\pi_t}\right) > (1-\rho)\,\beta E_t\left(\frac{\phi}{1+\pi_{t+1}}\right)+\rho\beta E_t\left(\frac{u'\left(\gamma_{t+1}+\delta\right)}{1+\pi_{t+1}}\right). \tag{52}$$

Again, since we are considering small fluctuations occurring near the steady state, it is sufficient to check that all four conditions hold with strict inequality in the steady state. As we show in Appendix B, this also occurs when δ belong to some non-empty interval: δ must be small enough to deter both euu and uuu households from saving for precautionary purposes, while at the same time being large enough to induce positive self-insurance by employed as well as eu-households. For the sake of simplicity, we carry out this analysis in the zero-inflation steady state, i.e., where $\gamma = 0$ (by continuity, our existence conditions will remain valid for small deviations of steady state inflation from zero.)

Aggregate dynamics. Substituting (50) into (46)–(47), one obtains a two-dimensional, nonlinear backward/forward dynamic system with forcing sequence $\{\tau_t\}_{t=0}^{\infty}$ and solution sequences $\{m_t^e\}_{t=0}^{\infty}$ and $\{m_t^{eu}\}_{t=0}^{\infty}$. Linearising this system around the zero-inflation steady state, we find a dynamics of the form:

$$E_t\left(\Gamma_1 X_{t-1}+\Gamma_2 X_t+\Gamma_3 X_{t+1}+\Gamma_4\hat{\tau}_t+\Gamma_5\hat{\tau}_{t+1}\right)=0, \tag{53}$$

where $X_t \equiv \left[\begin{array}{cc}\hat{m}_t^e & \hat{m}_t^{eu}\end{array}\right]'$ and where the Γ_j, $j=1,...,5$ are conformable matrices. As is well known from the literature on expectational linear systems (e.g., Uhlig, 2001), the solution to equation (53) has the following first-order autoregressive representation:

$$X_t = PX_{t-1}+Q\hat{\tau}_t, \tag{54}$$

where the 2×2 matrix P and the 2×1 vector Q can be recovered from (53) using the method of undetermined coefficients. With γ_t generated by X_t, X_{t-1} and τ_t (see Eq. (50)) and X_t given by (54), we can use Eqs. (40)–(42) to infer the labour supplies of employed households (i.e., l_t^{ee}, l_t^{uue} and l_t^{eue}), and then compute aggregate output using (48).

Panel B of Figure 1 plots impulse-response functions for the various variables of interest to a money growth shock, under the same parameter configuration as that used for the two wealth state model.[9] Although the overall output response to the shock looks similar to that in the four-agent economy, it (unsurprisingly) masks a lot more heterogeneity at the individual level; moreover, under our parametrisation the three-wealth state equilibrium exhibits a much larger output response to a normalised money shock than the two-wealth state equilbrium.

5.2 $n > 3$ wealth states

Extending the analysis to allow for more than three wealth states is straightforward (although possibly tedious for large values of n). First, conjecture the existence of an equilibrium where unemployed households become borrowing-constrained at the $(n-1)$th period of continuous unemployment but not before, while employed households are never borrowing-constrained. In this economy there are $n-2$ types of unconstrained unemployed households and 2 types of constrained unemployed households; we thus end up with n wealth states, once the (unique) end-of-period wealth level of the employed and the zero wealth level of the constrained unemployed have been included in the state space. With n types of unemployed households there is also n types of employed households: $n-1$ types corresponding to the different wealth levels with which unemployed households may end the period (with the two unemployed types ending the period with zero wealth generating only one type of employed household), plus one type corresponding to those households who were already employed in the previous period. An economy with n wealth states thus has $2n$ types of agents.

In this economy, the $n-1$ levels of positive real money holdings by the unconstrained types are generated by the $n-1$ Euler equations characterising the behaviour of those types. Formally, once the money market clearing condition is substituted into each Euler equation, one obtains a $(n-1)$-dimensional dynamic system for $n-1$ sequences of positive real balance levels that solve this system given the forcing sequence of money-growth shocks. The approximate VAR representation of the solution vector can be recovered by linearising this system and applying the method of undermined coefficient. This ultimately yields an expression similar to (54), but with X_t now being a $(n-1)$-dimensional vector of positive end-of-period real money holdings, expressed as proportional deviations from the steady state. Last, output is the weighted sum of the n levels of labour supply by the different

[9] Here we still impose that $\delta = 0.9.\delta_+$, but δ_+ is now given by (B3) in Appendix B, rather than (A2) in Appendix A. Then we check numerically that $0.9.\delta_+ > \delta_-$, and that the equilibrium generated by (53) is determinate and thus unique.

types of employed households, which are in turn determined as residuals from their budget constraints.

The conditions for the existence of an equilibrium with n wealth state are reasonably simple to check. First, δ must be sufficiently low so that in the steady state the real balances held by households who have been unemployed for $n-2$ periods is non-negative (i.e., these households are not borrowing-constrained); since unemployed households gradually deplete their wealth, this implies that all households which have been unemployed for fewer than $n-2$ periods are also unconstrained. Second, δ must be sufficiently large so that households are borrowing constrained at the $(n-1)$th period of continuous unemployment; since all households with more than $n-1$ periods of continuous unemployment hold zero beginning-of-period real balances (i.e., they hold strictly less than households having experienced only $n-1$ periods of unemployment), all such households are also borrowing-constrained along the n-wealth level equilibrium. That there always exists a range of δ satisfying both conditions follows from the fact that, for any value of δ and under the conjectured equilibrium, households entering their $(n-2)$th period of continuous unemployment have strictly greater beginning-of-period wealth than those entering their $(n-1)$th unemployment period.

6 Concluding remarks

This paper has modelled some dynamic and welfare effects of aggregate monetary shocks in a Bewley-type monetary model with idiosyncratic labour income risk. We have shown that money-growth shocks that contemporary redistribute real money wealth across agents tend to raise output (through the intratemporal inflation tax), but also that this direct effect can be offset by the effect of expected future redistribution on the real demand for cash (i.e., through the intertemporal inflation tax). Finally, the fact that wealth is redistributed both at the time of the shock and in the future (provided that money growth variations are persistent), combined with the perpetual transitions of households across employment statuses and cash-holding levels, implies that the welfare effects of monetary shocks are in general ambiguous (in the Pareto sense.)

The inherent complexity of Bewley-type models with both idiosyncratic and aggregate uncertainty, which results from the very large number of agent types that these models typically involve, is notorious and may have hindered their use (see the discussion in Kehoe and Levine, 2001). Our response to this challenge has been to construct, and then to focus on, a class of closed-form equilibria with a small number of wealth states and thus limited household heterogeneity. We considered the simplest class of equilibria featuring cross-sectional heterogeneity (i.e., equilibria with two wealth states and four agent types) to derive our results and discuss the two effects that ultimately determine the sign slope of the output-inflation tradeoff. We also showed how our framework could be extended to handle richer classes of equilibria whilst maintaining analytical tractability. The relative simplicity of our heterogenous agents framework may make it a useful tool for the better understanding of the effects of other macroeconomic shocks (e.g., fiscal policy shocks) in economies with incomplete markets.

Appendix A: Steady state of the two-wealth state model

We use variables without time indices here to indicate steady-state values. From Eqs. (2)-(3), steady-state inflation and real transfers are $1+\pi=\tau$ and $\gamma=m\pi/(1+\pi)$, respectively (and since $\tau_t>\beta\ \forall t$ by assumption, we have that $\tau>\beta$ and $\pi>\beta-1$.) Substituting these values into (15) and using (21), we find that the steady-state real money holdings of employed households, m^e, are:

$$m^e=\frac{1+\pi}{1+(1-U)\pi}\left(u'^{-1}\left(\frac{(1+\pi-\alpha\beta)\phi}{(1-\alpha)\beta}\right)-\delta\right).$$

The values of c^{uu}, c^{eu}, l^{ee}, l^{ue} and y are then straightforward to derive. For example,

$$c^{eu}=u'^{-1}\left(\frac{(1+\pi-\alpha\beta)\phi}{(1-\alpha)\beta}\right). \tag{A1}$$

Since we consider fluctuations which are arbitrarily close to the steady state, a sufficient condition for our closed-form solution to be an equilibrium is that both (22) and (24) hold with strict inequalities in the steady state. From (28), the first condition is simply:

$$\delta<u'^{-1}\left(\frac{(1+\pi-\alpha\beta)\phi}{(1-\alpha)\beta}\right)\equiv\delta_+. \tag{A2}$$

In the steady state, the left-hand side of (24) is c^{eu}. Using (A1), inequality (24) becomes:

$$\frac{(1+\pi)(1+\pi-\alpha\beta)}{(1-\alpha)\beta}-(1-\rho)\beta>\frac{\rho\beta}{\phi}u'(\delta+\gamma). \tag{A3}$$

In the steady state, $\gamma=(1-U)m^e\pi/(1+\pi)$ (see Eqs. (2) and (21)). Substituting γ into (A3), using Eq. (28) and rearranging, we may rewrite the latter inequality as:

$$\frac{(1+\pi)(1+\pi-\alpha\beta)}{(1-\alpha)\beta}-(1-\rho)\beta>$$
$$\frac{\rho\beta}{\phi}u'\left(\frac{\delta}{1+(1-U)\pi}+\frac{(1-U)\pi}{1+(1-U)\pi}u'^{-1}\left(\frac{(1+\pi-\alpha\beta)\phi}{(1-\alpha)\beta}\right)\right). \tag{A4}$$

The left-hand side of (A4) is positive at $\pi=\beta-1$ and thus for all $\pi>\beta-1$. The right-hand side of (A4) is decreasing and continuous in δ over $[0,\infty)$. Thus, if (A4) holds at $\delta=\delta_+$, then by continuity there exists $\delta_-<\delta_+$ such that (A4) holds for all $\delta>\delta_-$. Setting $\delta=\delta_+$ in (A4) and rearranging, we have:

$$(1+\pi-\rho\beta)(1+\pi-\alpha\beta)-(1-\rho)(1-\alpha)\beta^2>0, \tag{A5}$$

which is always true whenever $\pi>\beta-1$.

Section 4.1 referred to the comparative-static result that steady-state inflation decreases with mean inflation. To show this, note that a sufficient condition for $\partial m^e/\partial\pi < 0$ is that

$$\frac{\partial\left(1+\pi\right)u'^{-1}\left(\left(1+\pi-\alpha\beta\right)/\left(1-\alpha\right)\beta\right)}{\partial\left(1+\pi\right)} < 0.$$

Since from (A1) $u'^{-1}\left(\left(1+\pi-\alpha\beta\right)\phi/\left(1-\alpha\right)\beta\right) = c^{eu}$, this condition may be written as:

$$c^{eu} + \left(1+\pi\right)\phi/\left(1-\alpha\right)\beta u''\left(c^{eu}\right) < 0,$$

or, after rearranging, $\sigma\left(c^{eu}\right) < \left(1+\pi\right)/\left(1+\pi-\alpha\beta\right)$. This is always true since $\sigma\left(c\right) \leq 1\ \forall c$ by assumption.

Appendix B: Steady state of the three-wealth state model

Around the zero-inflation steady state we have that $\pi_t = \gamma_t = 0$ at all dates. The steady-state counterparts of Eqs. (46)–(47) are thus, respectively,

$$\phi = \alpha\beta\phi + \left(1-\alpha\right)\beta u'\left(m^e - m^{eu} + \delta\right),$$
$$u'\left(m^e - m^{eu} + \delta\right) = \left(1-\rho\right)\beta\phi + \rho\beta u'\left(m^{eu} + \delta\right).$$

Combining these two equations, we obtain the following steady-state levels of real money holdings by eu and employed households:

$$m^{eu} = u'^{-1}\left(\phi\left(\frac{1-\alpha\beta}{\left(1-\alpha\right)\rho\beta^2} + 1 - \frac{1}{\rho}\right)\right) - \delta \quad \text{(B1)}$$

$$m^{e} = u'^{-1}\left(\frac{\phi\left(1-\alpha\beta\right)}{\left(1-\alpha\right)\beta}\right) + m^{eu} - \delta \quad \text{(B2)}$$

Our existence conditions are that $m^e, m^{eu} > 0$ (for small shocks, this will ensure that $m^{eu}_t, m^e_t \geq 0$ for all t, as required), and that inequalities (51)–(52) hold in the steady state. From (B1), $m^{eu} > 0$ if and only if:

$$\frac{\left(1-\alpha\beta\right)}{\left(1-\alpha\right)\rho\beta^2} + 1 - \frac{1}{\rho} < \frac{u'\left(\delta\right)}{\phi}. \quad \text{(B3)}$$

Note that if (B3) holds, then it is also the case that $m^e > 0$, since from (B2) we have that $m^e > m^{eu}$ if and only if

$$\frac{\left(1-\alpha\beta\right)}{\left(1-\alpha\right)\beta} < \frac{u'\left(\delta\right)}{\phi},$$

which is always true under condition (B3).

Since $c_t^{euu} > c_t^{uuu}$ we have that $u'(c_t^{uuu}) > u'(c_t^{euu})$. Thus, if inequality (51) holds then so does (52). The steady-state counterpart of (51) is:

$$u'(m^{eu} + \delta) > (1 - \rho)\beta\phi + \rho\beta u'(\delta),$$

or, using (B1) to substitute m^{eu} out and rearranging,

$$\frac{(1 - \alpha\beta)}{(1 - \alpha)\rho^2\beta^3} + \left(\frac{\rho - 1}{\rho}\right)\left(\frac{1}{\rho\beta} + 1\right) > \frac{u'(\delta)}{\phi}. \tag{B4}$$

Inequality (B3) defines δ^+, the maximum amount of home production above which eu households would no longer find it worthwhile to save. Inequality (B4) defines δ^-, the minimum amount of home production below which euu households would prefer to save, rather than borrow. There exist a range of δ satisfying both (B3) and (B4) if and only if the right hand side of (B4) is larger than the left hand side of (B3). After rearranging, this turns out to be the case if and only if:

$$(1 - \rho\beta)(1 - \alpha\beta) - (1 - \rho)(1 - \alpha)\beta^2 > 0, \tag{B5}$$

which is always true (note that inequality (B5) is equivalent to (A5) with $\pi = 0$.)

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